SYDE 312 UNIT 5: ODE SAMPLE PROBLEMS

These problems were from a different text, the one used in the 2004 class. The solutions may be helpful.

Problem 1

Initial value problem is:

$$y' = x + y - xy, y(0) = 1$$

The taylor series is:

$$\begin{aligned} y(x) &= y(x_0) + y'(x_0)h + \frac{y''(x_0)}{2!}h^2 + \frac{y'''(x_0)}{3!}h^3 + \cdots \\ y_5(x) &= 1 + x + \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{6} \end{aligned}$$

At $x = 0.1 : y(0.1) = 1.104817$
At $x = 0.5 : y(0.5) = 1.593750$

Using the ode23 and ode45 functions:

```
function yp = eqnfile(x,y)
yp = x+y-x*y;
>>[t, y]=ode23('eqnfile', [0 0.1], 1)
y(0.1)=1.1048
>>[t, y]=ode23('eqnfile', [0 0.5], 1)
y(0.5)=1.5942
>>[t, y]=ode45('eqnfile', [0 0.1], 1)
y(0.1)=1.1048
>>[t, y]=ode23('eqnfile', [0 0.1], 1)
y(0.5)=1.5942
```

Problem 5

Initial value problem is:

$$y' = x + y - xy, y(0) = 1$$

Simple Euler method:

$$y_{n+1} = y_n + hf(x_n, y_n)$$

With $h = 1/2^{15} : y(0.1) = 1.10482$
With $h = 1/2^9 : y(0.5) = 1.59384$

Problem 7

IVP is:

$$y' = x + y - xy, y(0) = 1$$

Modified Euler method:

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_{n+1})]$$

With $h = 0.00525 : y(0.1) = 1.10482$, and $y(0.5) = 1.59420$

Problem 8

IVP is:

$$\dot{y} = y^2 + t^2$$

Analytically: $y(2) = 6.70383$

Modified Euler method:

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_{n+1})]$$

With $h = 0.1 : y(2) = 6.15633$
With $h = 0.5 : y(2) = 6.51880$
Actual error at $h = 0.5 : 6.70383 - 6.51880 = 0.18503$
Estimate of error is 0.12082

Using ode23: $y(2) = 6.6823$
Using ode45: $y(2) = 6.7037$

Problem 18

$$y' = x + y - xy, y(0) = 1$$

Fourth-order Runge-Kutta method:

$$y_{n+1} = y_n + \frac{1}{6}g_1 + \frac{1}{3}g_2 + \frac{1}{3}g_3 + \frac{1}{6}g_4$$

$$\begin{aligned} g_1 &= hf(x_n, y_n) \\ g_2 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}g_1) \\ g_3 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}g_2) \\ g_4 &= hf(x_n + h, y_n + g_3) \end{aligned}$$

Repeating the iterations based on different values of step size $h = 1/2, 1/4, 1/8$ and so on, we arrive at $y(1.0) = 2.194955$ when $h = 1/8$. This step size is sufficient to get the stated value for $y(1.0)$.

Problem 19

$$y' = \frac{1}{x+y}$$

Analytically: $y(1) = 2.377974$

R-K4 method iterations are:

$$y_{n+1} = y_n + \frac{1}{6}g_1 + \frac{1}{3}g_2 + \frac{1}{3}g_3 + \frac{1}{6}g_4$$

$$\begin{aligned} g_1 &= hf(x_n, y_n) \\ g_2 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}g_1) \\ g_3 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}g_2) \\ g_4 &= hf(x_n + h, y_n + g_3) \end{aligned}$$

With step size of $h = 0.2 : y(1) = 2.377974$.

Problem 43

The system of first order ODEs is:

$$\begin{aligned}\frac{dx}{dt} &= xy - t \\ \frac{dy}{dt} &= x + t\end{aligned}$$

Initial conditions are $x(0) = 1, y(0) = 0$.

To solve this with Matlab we can relabel the system like this:

$$\begin{aligned}\frac{dy_1}{dt} &= y_1 y_2 - t \\ \frac{dy_2}{dt} &= y_1 + t\end{aligned}$$

Initial conditions are $y_1(0) = 1, y_2(0) = 0$. The functions $x(t)$ and $y(t)$ are now written in vector form $y_1(t)$ and $y_2(t)$ respectively.

We can implement this easily in Matlab now. You set up an inline function f(t,y) where y is your vector of unknown solution functions $[y_1;\ y_2]$. Note that f has to be an inline COLUMN vector function:

```
f=inline('[y(1)*y(2)-t ; y(1)+t]','t','y')
```

The last two things in the above define the arguments for the vector function f [see help inline]. Now to solve the IVP you use:

```
ode45(f,[0 1],[1;0])
```

Here the [0 1] is the range of t-values over which the solution is to be found; [1;0] is the column vector of initial values for your solution. This produces the plot shown in the figure below.

If you want to get the values, you use:

```
[t,y]=ode45(f,[0 1],[1;0])
```

At $t = 1.0$, using ode45 with abstol set to 1E-6 using odeset, I get:
$y_1(1.0) = 1.238471, y_2(1.0) = 1.552422$

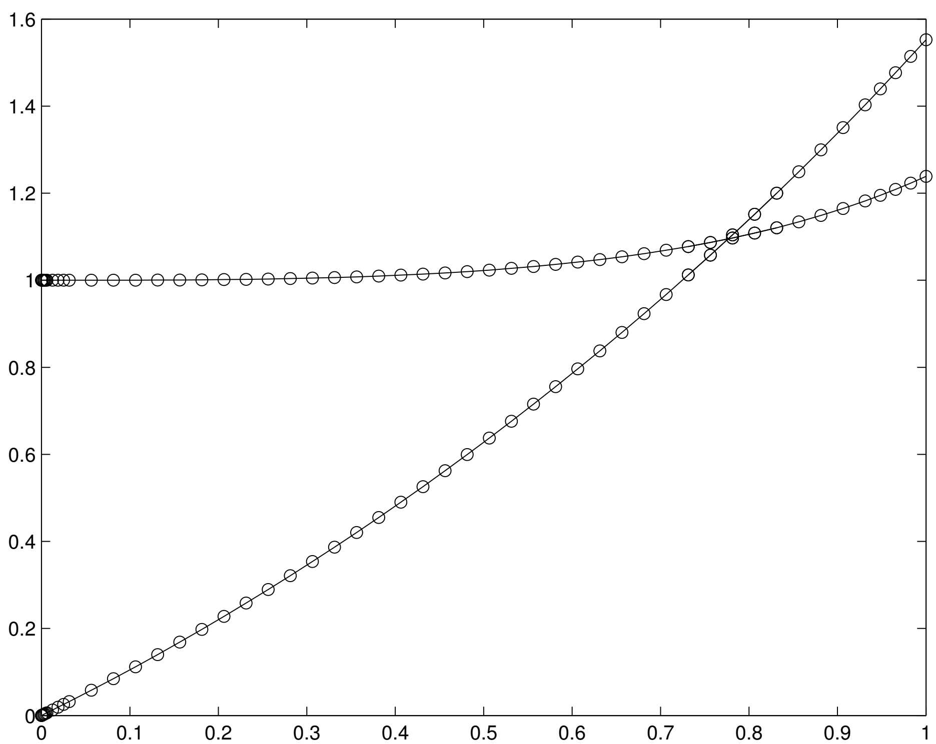

Figure 1: Problem 43: x(t) and y(t)