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Why Didn't Historical Makers Need Drawings?

Part II – Modular Dimensions and the Builder's *Werkzoll*

INTRODUCTION

In order to reconstruct the design basis of an historical artefact such as a musical instrument a proper historical perspective is essential. Plausible, but erroneous, conclusions can follow all too easily from *a priori* assumptions which are either extraneous to the instrument being examined, or which are based on a-historical reasoning that is only 'obvious' in a modern context.

In the first part of this article we examined how an historical builder used practical geometry to achieve correct proportional relationships in design. Geometric techniques circumvent the need to manipulate numerical quantities via algebraic calculation, by using physical manipulation of geometric figures, or through simple approximations of these. Architectural thinking was central to all the crafts, and provided the practical technique of modular design. Proportional relationships between components were determined geometrically from a generating dimension called the *module* which also fixed the absolute size of the whole design. As the architectural works make abundantly clear, the size of the module was essentially arbitrary, and was often derived as a multiple of a more basic elementary modular unit which we call the *modular inch*.

This article examines how the modular inch is likely to have been used by a builder of stringed keyboard instruments. The results suggest a procedure by which an accurate modular inch for a particular extant instrument may be determined with some confidence. The size was determined primarily by practical utility. In general, no *a priori* connection between a modular inch and any local unit of measure can be assumed, however this may have been the case where dictated by convenience (e.g. in response to out-sourcing of work and the inter-dependence of the crafts), or by some statute or regulation which forced a particular professional standard measure to be used by a group of builders. In general, a modular inch need not have been applicable anywhere but in a specific workshop, and to emphasize this it may also be called the builder's *Werkzoll* ('work inch').

In historical design practice, a modular design was achieved in its purest form through direct constructive geometry, with the module as the only independently measured dimension. This procedure relied on practical geometry using the well-known craftsman's layout tools – the square, compasses (including the large beam compass or trammel), and straightedge – to construct every dimension from the generating module. A detailed example of this type of layout procedure, for five-octave Viennese pianos of the school of JA Stein, was described in the first part of the article.

An alternative geometric practice, which we call *modular measurement*, involved the transfer of *modular dimensions*, i.e. simple multiples of half a modular inch, to construct the design following a prescribed formula. What is gained with this approach in terms of simplicity is perhaps lost by an increased abstraction. Pre-determined nominal modular dimensions were used to define the sizes and spatial arrangement of the parts in the design. Sequences of modular dimensions might also be considered to imply (when interpreted as rational approximations) geometric constructions, but such considerations were theoretical and it would not have served a practical purpose to think in those terms.

These two pure forms of modular design – constructive geometry and modular measurement – are the extremes of a spectrum, and, in reality, a mixture of the two techniques was likely to have been used. Regardless of the approach taken by a builder, many arbitrary dimensions which could be chosen independently, and did not contribute to the holistic geometry of the design were also likely to have been derived for simplicity as nominal modular dimensions. In this way, the modular inch can be expected to pervade the design of a stringed keyboard instrument, and it is the job of the analyst to reconstruct its size by working backwards from the whole.

We begin by examining fundamental issues related to the collection and analysis of dimensional data from extant instruments. This leads to a discussion of the elusive concept of acceptable tolerance for comparison of proposed nominal dimensions with actual observed dimensions. This problem is easier to deal with for direct geometric construction, which naturally produces, very accurately, internally self-consistent relationships. Any inaccuracy in the single independent generating dimension will be reflected in all subsequent dimensions in the instrument. On the other hand, modular measurement is intrinsically prone to inaccuracy. Moreover, the independence of individually transferred modular dimensions implies that these dimensional deviations from nominal modular sizes are also independent and it will be more difficult to eliminate the effects of random variation to provide evidence in support of a proposed nominal design scheme based on modular measurement.

These analytical problems are exacerbated by any uncertainty about the size of the modular inch on which the dimensions are based. In order to address this uncertainty, this article focuses on an examination of the problem of how to reconstruct the modular inch value used for a particular extant instrument. A large database of instruments is essential to eliminate random variation from nominal dimensions and make meaningful conclusions in this context. The results presented here are based on an analysis of more than 30 instruments of various types and periods. By considering how a builder is likely to have used the modular inch we can make conclusions about how the inch will be manifest in an instrument. A variety of independent design aspects are suggested in this regard, including string spacing and structural width for parallel instruments (i.e. flügel-shapes), and, for all types of stringed-keyboard instrument, keyboard dimensions, long case dimensions, and string lengths and small case dimensions to a lesser extent. The *a priori* assumption that a particular layout method was used, for instance modular measurement without due consideration being given to the possibility of constructive geometry, or perhaps both methods in some combination, can lead the analyst astray. By considering all the proposed modular design aspects simultaneously, the chance fit of observations around several conflicting geometric layout methods with quite different modular inch sizes can be minimized. Above all else, it behoves the analyst to maintain at all times a level of pragmatism which matches that of the historical builder.

DATA FROM EXTANT INSTRUMENTS

Apart from the Arnaut manuscript,[1] there is unfortunately very little relevant source material relating explicitly to the practice of specific instrument builders. Therefore, attempts to unravel the procedures which led to a design must rely on the extant instruments themselves as the primary source of information. The situation with this sort of analysis is far from ideal. As Denzil Wraight succinctly describes the problem: 'Finding an explanation which fits the instrument is no guarantee that the explanation is correct.'[2] This implies that analysis should be approached in terms of establishing a weight-of-evidence argument, which is typically used as sufficient proof for propositions which cannot be proven directly. In fact, this approach is the fundamental basis

[1] Henri-Arnaut de Zwolle. Fifteenth-century Burgundian manuscript. A complete English translation of Arnaut's instructions for constructing a clavisimbalum (harpsichord) and other stringed keyboard instruments is given in: Stewart Pollens. *The Early Pianoforte*. Cambridge Univ. Press, 1995. Chapter 1. The full text of the Arnaut manuscript is available in modern reprint and French translation in: G. le Cerf, Ed. *Les traités d'Henri-Arnaut de Zwolle et de divers anonymes*. Paris: Editions Auguste Picard, 1952.

[2] D. Wraight. The identification and authentication of Italian string keyboard instruments. In: H. Schott, Ed., *The Historical Harpsichord*, Vol. 3. Stuyvesant, NY: Pendragon Press, 1992, pp. 59-161 (see page 75).

for the scientific method in which theories can never be proved, only disproved. It is easy enough, however, to arrive at an anachronistic, or impractical solution, in the reconstruction of historical methods from extant instruments. Circumstantial evidence must be used very carefully, with an appropriate mix of discipline and common sense.

A geometric design practice implies that at least a few specific points in an instrument must have been explicitly located by a builder during the layout process. Essentially arbitrary points that were located by simple modular measurement need not have been marked out with any great accuracy. In contrast, points which were located using a trammel or compasses, directly from others already present, would automatically have been located highly accurately. In particular, proportional relationships between dimensions in a specific instrument laid out with a trammel construction can be expected to be the same as those of similar instruments which followed the same geometric scheme.[3] From these considerations, it can be concluded that, to determine the use of a construction method in which potentially inexact points were located with a modular scale (e.g. the Ruckers school), a considerable database from extant instruments is required to deal with the inevitable variation in observed dimensions; a comparatively smaller database will suffice to establish confidently the highly accurate trammel-based geometric constructions used by some builders (e.g. the Stein fortepiano school).[4]

Variation and discrepancy between a proposed theory and observed dimensions can also be attributed to other factors. Given the age and condition of many extant instruments it might be suspected that the initial special construction points will not be in their original locations, perhaps due to wear and tear, or wood shrinkage. This is not generally a problem for the dimensionally stable longitudinal dimensions. Furthermore, lateral dimensions are often constrained by members such as braces or a bellyrail glued across them, in which case they, too, will be close to their original specifications, even if there has been cross-grain shrinkage of the board on which they are marked (resulting in cracks). The scenario of a builder marking out a bottom and allowing a period of time to elapse before constraining the cross-grain direction cannot be dismissed, and this could potentially introduce dimensional changes which cannot be differentiated from erroneous theory in the analysis of an extant instrument. Case distortion must also be given careful consideration, since it can alter the apparent geometry. A distorted case with a still relatively flat bottom will introduce only marginal changes to a geometric layout. If bottom distortion has occurred, an appropriate correction must be made before geometric analysis can be completed effectively. When using drawings of such instruments, it is important to have unambiguous information on the exact current configuration – idealized drawings are of little use for geometric analysis.[5]

When it is possible to gain access to the inside bottom surface of an instrument, one can occasionally find the builder's actual construction marks. However, in general it cannot be expected that these should be still visible, due to the age of the instrument, and also because it is likely, for the most part, that they will have been obscured by subsequent construction, as parts were glued onto the bottom boards, or edges were planed off to meet a scribed line. For instruments in which the bottom is only loosely attached, such as Ruckers harpsichords, meaningful scribe lines can be observed on many instruments.[6] Many of these are probably there to indicate the location of components such as braces, for finding the nailing positions, so care must be taken to determine which of the various observed lines were actually used as part of a geometric construction and which may be there simply for convenience in assembly. Italian

[3] The absolute size of the instrument may vary, but not the proportional relationships.

[4] S. H. Birkett and W. Jurgenson. Why didn't historical instrument makers need drawings? Part I – Practical geometry and proportion. *Galpin Society Journal* LIV: 242-284.

[5] The published technical drawing of the JL Dulcken *c.*1795 piano is an excellent example of the presentation of accurate information which includes full details of existing distortion. Smithsonian Institution, Inv. No. 303,537. Drawing by T. Wolf et al, 1975. (See the Appendix for details of this and other instruments referred to in the text.)

[6] G. O'Brien. *A Ruckers Harpsichord and Virginal Building Tradition*. New York: Cambridge Univ. Press, 1990.

harpsichords often show layouts scribed on the bottom, including sometimes positions of bridge(s), nut(s) and wrestplank. Hubbard, for example, reports that he has seen 'one Italian harpsichord in which the maker had drawn the plan view of the instrument in full scale on the inside of the bottom.'[7] Many early virginals have visible scribed lines on the soundboard surface (Figure 1). Any of these builders' marks may be useful, but they should be approached with some caution before drawing conclusions from them.

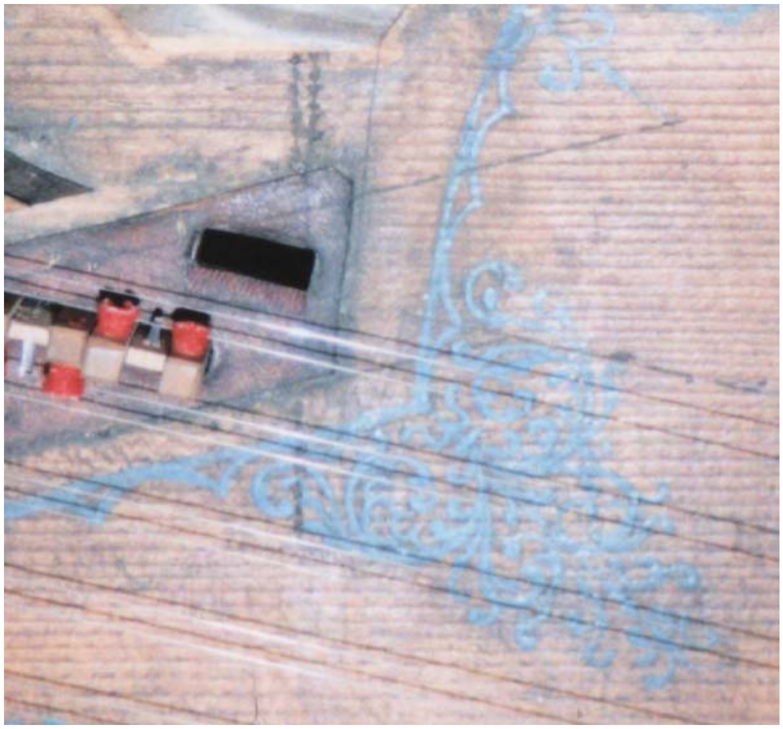

Figure 1. *Treble end of register of a virginal by van der Biest, showing scribe lines made by the maker.*

When recording data for geometric analysis from an extant instrument, it is important that all required dimensions are measured directly. Relying on general, non-specific dimensional data, and calculating the required values, is insufficiently accurate and inadequate for testing geometric theory. For instance, as will subsequently be demonstrated, *Stichmass* (i.e. standard three-octave span) is a modern invention which is meaningless from a builder's point of view, yet this is generally all that is reported, rather than the useful overall width of the keyboard, the space into which it fits, and the dimension of the keyboard/action frame. Case, and (or) bottom, outlines should always be taken using triangulation methods to minimize error and provide highly accurate geometric data. This precludes the use of most of the previously collected data published in catalogues and books. Traditional organological data generally pertains to the outside of outer cases (excluding only mouldings), making it irrelevant for most constructions based on the bottom, which generally coincides with the inside of the outer case in such instruments.[8] String

[7] F. Hubbard. *Three Centuries of Harpsichord Making*. Cambridge, Mass: Harvard Univ. Press, 1965, pp. 210-11

[8] This point has also been made recently in: G. O'Brien. 'The use of simple geometry and the local unit of measurement in the design of Italian stringed keyboard instruments: An aid to attribution and to organological analysis.' *Galpin Society Journal*. LII (1999) 108-171. In some instrument building traditions in which the case was not built on the bottom, for instance that of the Ruckers, it is actually the

spacing, if given, is invariably reported in general terms, however, to be useful, precise information about localized variation in spacing is also required. Generally the best data is obtained directly from the spacing on the rack (e.g. damper housing, keytail guides, etc.), but this information is seldom provided in published descriptions of an instrument.

Given the extensive personal effort required to examine and measure an instrument for geometric analysis, it is inevitable that technical drawings must be consulted to expand the database to a reasonable size. This can be satisfactory, provided the precise original configuration of the instrument is deducible from the drawing. A dimensionally-stable plastic medium is preferable, since paper drawings frequently change size unpredictably. The latter problem can be corrected by proportional adjustment, if a scale is provided on the drawing and checked in the two orthogonal directions. In most cases only minor change in paper size is observed (a ratio of 1005:1000 is not unusual though), but, as for wood, this often does not occur isotropically, so the aspect ratio,[9] and therefore proportional relationships, may be altered. The most effective way to deal with a paper drawing is to convert it to a computer drawing in a CAD program, transferring dimensions by triangulation, and apply a proportional scaling as appropriate in each direction. All conclusions about absolute dimensions taken directly from a paper drawing are subject to correcting the scale for printing distortion and (or) paper shrinkage and expansion.

ACCEPTABLE TOLERANCE

To infer with confidence that a particular geometric construction is likely to have been used in the design of an extant instrument demands a close agreement between observed and predicted dimensions. For trammel-based methods of constructive geometry, the nature of the layout tool suggests that tolerance should be independent of the absolute dimension being transferred, i.e. a fixed value rather than a percent. A tolerance as small as 1mm would be typical for points constructed with a trammel, even on dimensions as large as 2000mm, giving a remarkable accuracy of 0.05%. This is considerably less than the tolerances usually accepted in analyses of musical instruments.[10] Correct analysis in this context demands only a high degree of internal consistency, therefore some variation in absolute dimensions is acceptable for instruments that are supposed to be of the same design. The actual size of the module may not always agree perfectly with its nominal value, but direct constructive geometry ensures that, within an instrument, all the generated relationships will be accurate. This implies that proposed geometric constructions of this type can generally be accepted or rejected with some confidence, since the agreement between theoretical and observed dimensions is established so closely that coincidence is highly unlikely.

The situation is less satisfactory for instrument designs based on direct modular measurement. As Denzil Wraight puts it, ‘the unlikelihood of the accidental occurrence must be seen as a most deceptive criterion.’[11] Historical builders worked quickly and some sloppiness must inevitably have occurred for dimensions that were not important, especially so for high output shops such as those of the Ruckers. To circumvent this analytical problem requires a wider tolerance for acceptability, and consequently a wider base of extant instruments must be considered, to permit a proper statistical treatment of the dimensional variation. For example, it is impossible to devise a single scheme that predicts, to a 1mm tolerance, the precise geometric configuration of even just the Ruckers single manual harpsichords from a particular shop, because there is so much variation in the dimensions. Nevertheless, these builders certainly must have worked following *some* nominal scheme, and the purpose of analysis is to reconstruct that geometry, rather than to predict the (random) variation observed in specific instruments. This

outside case dimensions which are relevant to the geometry. The JL Dulcken *c.*1795 piano (see Appendix) was laid out on a bottom which extends to the outside of the outer case.

[9] The ratio of length to width.

[10] For example: K. Coates. *Geometry, Proportion and the Art of Lutherie*. Oxford University Press, 1985; H. Heyde. *Musikinstrumentenbau.* Wiesbaden: Breitkopf und Härtel, 1986.

[11] Wraight, op. cit., p.76.

places a heavy responsibility on the analyst, and demands a highly disciplined, historically focused approach.[12]

Acceptable tolerance is inextricably linked to analytical tactics. '... How easy it is to work over an undigested mass of data and emerge with a pattern, which at first glance, is so intricately put together that it is difficult to believe it is nothing more than the product of [the analyst's] brain. Consciously, or unconsciously, their preconceived dogmas twist and mold the objective facts into forms which support the dogmas, but have no basis in the exterior world.'[13] This bending of the facts to fit a theory can be minimized by using highly accurate and specific data, and a small tolerance. Even so, there are complications. For instance, in considering a proposed scheme for string scaling, Wraight asks 'how large a deviation of a string length from its intended length would be significant?'[14] Even this question itself makes the *a priori* assumption that string lengths were indeed explicitly measured by a builder. It is quite plausible that some builders could have implemented a general idea of suitable scaling by explicitly locating the edge of the bridge, rather than the location of the string termination points. Even string lengths measured at the time of laying out an instrument would have been subject to *ad hoc* adjustments during the subsequent construction, and possibly random variation from inaccuracies as well. In this regard, different types of stringed instruments were built by different builders and schools with different concepts of acceptable variation in nominal scaling, i.e. desirable string lengths, so it is impossible to provide a general answer to Wraight's question. Complications such as these can usually be associated with a focus on details, combined with some *a priori* assumptions on the part of the analyst. The difficulties can generally be avoided by adopting an holistic perspective which properly reflects the historical builder's approach to design and construction, rather than becoming tied up with details and specifics.[15]

Acceptable tolerance can also be bought at the expense of simplicity. The overall structure and inter-relationships that govern the location, geometry, and dimensions of all the component parts of an instrument can be expected to be simple, elegant, and pragmatic. That is the nature of historical practice. Complex solutions should be viewed with suspicion. Any sufficiently complicated theory, or an adequately large collection of related theories, can usually be forced into excellent agreement with a specific set of observations. For example, Coates was able to achieve a rather small tolerance[16] in his analyses of stringed instruments, simply by using a large enough base of potential proportional schemes from which to choose his proposed theories. 'It will always be possible to analyse these sorts of shapes in the way that has been attempted because of the amount of inaccuracy allowed for, especially when different perceptions of the various shapes are employed in order to maximize the apparent richness of the mathematical analyses.'[17]

The illusion of acceptable tolerance can be created by making the right *a priori* assumptions. However, theories based on forcing a-historical, or simply incorrect, concepts to fit the design of

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[12] See the discussion of this point in: R. Gug. 'Geometry, lutherie and the art of historiography', *Fellowship of the Makers and Restorers of Historical Instruments* (*FoMRHI*) No. 59 (April 1990), pp.40-72.

[13] M. Gardner. *Fads and Fallacies in the Name of Science*. New York: Dover, 1957. Chapter 15. p.184.

[14] Wraight, op. cit., p.87.

[15] This can be contrasted with the comments in: J. Koster. 'Toward the reconstruction of the Ruckers' geometrical methods'. In: C. Rieche, Ed. *Kielinstrumente aus der Werkstatt Ruckers – zu Konzeption, Bauweise und Ravalement sowie Restaurierung und Konservierung: Bericht über die internationale Konferenz vom 13-15 September, 1996 im Händel-Haus, Halle*. Halle, 1998. The author advocates a reductionist approach, saying that 'no attempt will be made to present a complete geometrical analysis of any Ruckers instrument,' and suggesting that it is 'more productive to discuss general principles, illustrated by some examples.' We take a contrary view in that it is impossible to make any meaningful attempt at analysing a complete instrument based on a collection of ad hoc and unrelated procedures.

[16] Coates, op. cit. The stated tolerance of 1mm must be considered in relation to the size of the instruments he is analysing, mainly violins, lutes and so on. The author relaxes the tolerance when convenient to establish a desired theory.

an historical instrument often break down through self-inconsistency, or when they are extrapolated to other instruments, requiring exceptions to already complicated rules to explain the new context.[18] This problem is illustrated, for example, by a recent derivation of the unit(s) of measure which are supposed to have been used by the Ruckers family. The method is based on drawing an association between the lengths of extant, unaltered Ruckers virginals and their nominal lengths which were expressed as 'five-voet', 'six-voet' etc. [19] In theory this technique might provide the size of the voet (foot) unit of measurement used by the Ruckers. However, consideration of the remarks of Klaas Douwes, a near-contemporary to the Ruckers, immediately brings into question the basic *a priori* assumption on which the method is based: 'But I should say such clavecimbels as are called six-voet are not fully six voet long, but are about a third of a voet shorter. Then similarly, the 5-, 4-, and 3-voet also have not the full length, but are normally a bit shorter.'[20] This account certainly suggests that an analyst should proceed with caution when using nominal lengths for any sort of metrological calculation.[21] Nevertheless, the average lengths of these instruments are indeed seen to be very close to multiples (2½, 4, 4½, 5, and 6 voet) of a foot measure with a value of 285mm. This result is actually not inconsistent with Douwes' comments, provided his voet measure was 6 to 7 percent larger than this, i.e. in a ratio of about 6 to 5 .[22] The 285mm voet suggested by this analysis is itself very interesting,[23] and we will return to consideration of it in a later section, however no valid conclusion can be made about the size of the associated duim, which would require either knowledge of the number of duimen in one voet, or additional independent data to allow the duim value to be separately estimated. Since the latter is not available, O'Brien made the further *a priori* assumption that the voet was subdivided into 11 duimen, a common practice for contemporary Flemish units of measurement: 'I have taken the length in millimeters of each extant unaltered virginal and divided this by the nominal length in duimen. For the 6-voet instrument, for example, this means that the length in millimeters is divided by 66 duimen.' After consideration of the various different lengths, he concludes that a duim of size 25.88mm was used by the Ruckers. Again, the need for an important and critical *ad hoc* assumption suggests some caution is required when considering the results. In particular, even if we ignore Douwes' comments, only an estimate for the size of the voet can be supported by the virginal length data itself, because all the dimensions are supposed to be multiples of a voet and this information does not relate to the duim. No inconsistency would have been obtained, for example, if an assumed duim of 1/12, 1/20, or any other fraction of a voet, had been chosen instead of 1/11 voet. Each one of these assumptions would have given the same voet with a different duim value. The situation deteriorates when it is 'observed' from examination of extant Ruckers instruments that dimensions less than about one voet (e.g. case heights, distances between scribe lines, component sizes etc.) are integral or half-integral multiples of a 'slightly

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[17] M. Fleming. 'Some strangeness in the proportion' *FoMRHI Quarterly*, No. 50 (January 1988) Comm. 854, 46-48. In his attempts to reconstruct the designs proposed by Coates, Fleming found the actual tolerance is often 2mm, and that this is insufficient to differentiate between some of Coates' supposedly different proportional schemes.

[18] See, for example, the complex methods proposed in Heyde, op. cit.

[19] O'Brien, 1990, op. cit., pp.284-285. *Voet* is the Flemish term for a foot.

[20] Klaas Douwes. Grondig Ondersoek van de Toonen der Musijk. Franeker, 1699. Facsimile reprint, Amsterdam, 1970. Text also reproduced in: O'Brien, 1990, op. cit., p.293.

[21] Future metrological historians will go similarly astray if they attempt to derive twentieth-century units from nominal specifications of colloquial usage, e.g. 6 GB hard-drive, a 9 foot concert grand piano, North American nominal lumber sizes, a 17 inch computer monitor, a 110 V electricity system, a 5 day work week etc.

[22] Douwes also mentions a typical c2 scaling as 'l4 duimen' for a 6-voet virginal, i.e. at reference pitch, which is observed to be about 356mm on extant Ruckers instruments. Consistency between these two independent observations is possible only if Douwes was using a twelve inch English foot (304.8mm) for the unit of measurement in his article.

[23] This is presumably the 'Flemish voet' of which two are said to have a lengrh of 569mm in O'Brien, 1990, op.cit., p.82, even though it is a calculated value rather than one obtained from an historical source.

smaller duim.' This 'discrepancy in the measurement of long and short distances' is resolved by a further assumption that there were 'two standards of length in use in the Ruckers workshop at the same time': the 'small duim' (25.48mm) and the 'large duim' (25.88mm). Most of the dimensions cited in the book are given in mm, then expressed in terms of integral or half-integral multiples of either a large or small duim,[24] presumably to suggest that these dimensions were measured by the builder and to provide support for the nominal values determined. However, the availability of two close sets of duimen, provides sufficient flexibility that only a moderately relaxed acceptable tolerance is required to express almost any dimension reasonably well as 'about a half-integral multiple' of one of them Unfortunately these small and large duimem seem to have become organological standards, and are often used without even a reference to their origin.[25] Further complications have been added recently with yet another proposed Ruckers duim measure, the 'great duim' (26.2mm), suggested by Koster,[26] based on an observation by Heyde.[27] This metrological house of cards is not compatible with the fundamental nature of historical practice, viz. simplicity and pragmatism.We will return to an examination of Ruckers practice after developing the analytical methods in the following sections.

THE WERKZOLL

The modular approach to design was described by the architectural historian Rudolf Wittkower in reference to the Italian Renaissance: 'Every part in a building down to the minutest details has its fixed size and shape ... A building was an organic whole, completely definable in terms of metrical relationships.'[28] This description could just as well serve as the methodology behind historical instrument design, for which the module is the fundamental concept that ties together the geometry of the whole and its components. The elementary modular unit on which a particular instrument was based can accurately be called the builder's modular inch (or *Werkzoll*) for that instrument, since it was nothing more than a personal, shop-specific reference dimension. As practicality and convenience primarily governed the choice of module, no fixed *a priori* relationship with a local unit of measure can be inferred. This does not preclude the possibility that a builder may have used a local unit to define his module, however such connections are often tenuous.

It has been suggested that determining the modular inch used in an extant instrument, and comparing this to a library of historical local units of measure, is a feasible technique for attributing unsigned instruments to particular geographical regions, or even identifying the unknown builder.[29] According to O'Brien,[30] 'it is quite clear that any maker of musical instruments – or any other object for that matter – would have worked on a day-to-day basis using ... his local unit of measurement.' Unfortunately, although the idea expressed here is appealing, and, of course, some builders did use local units, historical reality contradicts any generalization in this regard. If this were *always* the case it would imply a one-to-one relationship existed between builders, modules, geographical/political regions, and local units of measure, which is clearly impossible on account of the continual movement of craftsmen between locales, apprenticeships which may have been served in more than shop, and especially because of the journeyman tradition, during which period a craftsman would have worked in no fixed locale.

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[24] O'Brien, 1990, op. cit.
[25] For example: Koster, 1998, op. cit., p.5. O'Brien's 'small duim' is called a 'short duim' with no reference to the origin of the measure.
[26] J. Koster, 2000, op. cit. 'Cathedrals, cabinetmaking and clavichords'. Part I. *Clavichord International* 4 (2000) 6-12, p. 6.
[27] Heyde, op. cit., p.161-2.
[28] R. Wittkower. *Palladio and Palladianism.* New York: George Braziller, 1974, p.64.
[29] Heyde, op. cit.
[30] G. O'Brien. The use of simple geometry and the local unit of measurement in the design of Italian stringed keyboard instruments: An aid to attribution and to organological analysis. *GSJ* LII (1999) pp.108-171, p.112.

Consequently, observed metrological associations between a builder's modular inch and a library of historical local measures must be of limited value.

To illustrate this point, before he adopted the standard Viennese *Zoll* of 26.33mm after moving to Vienna sometime about 1776, Anton Walter used a larger modular inch of 26.70mm which does not agree with any obvious local unit. This can be contrasted with the practice of Nannette Streicher, who apparently continued to use, throughout her long career in both Augsburg and Vienna, the modular inch of 25.91mm which characterizes the post-1783 instruments of her father, J. A. Stein. In particular, this means that for some 30 years Streicher instruments made in Vienna were based on a unit of measure different from the very well-established local one. The Stein modular inch also appears to have been used by other builders such as Ferdinand Hofmann and Louis Dulcken, working in Vienna and Munich, respectively. Moreover, this modular inch does not coincide with any known local German measure, and, in particular, it is different from the local measures in the areas where all these builders lived and worked. In a later section a possible explanation is given for the origin of this 25.91mm inch as a derived unit of measure which would have had no relevance beyond the piano-making craft.

The tenuousness of metrological connections can be illustrated further by Gottfried Silbermann, who used a modular inch of 26.70mm (320mm foot) in the design of the Silbermann 1749 piano.[31] Gottfried worked most of his life in the Freiberg area where the local Saxon inch was 23.6mm, again showing no apparent connection between the builder's modular inch and the local inch. Given the extent of geographical movements, and inter-connections and interactions between the careers of contemporary builders, this is not surprising. Gottfried's training was in Strassburg, with his older brother, Andreas, who had himself apprenticed with Casparini in Goerlitz. Goerlitz and most of Silesia is now in Poland, and was also Polish in medieval times, but Silesia was Bohemian (Austro-Hungary) during the period in question (1675-1740 when it became Prussian) and Saxon immediately before that. Thus the connections that brought Casparini back (in fact he may have never actually been outside of Austria since that then included Dalmatia and Croatia and Friaul bordering directly on Venetia) from Italy to Goerlitz, and Silbermann to him, are really quite close. After leaving Goerlitz, Andreas spent many years in Paris with the French builder Thierry, settling in Strassburg and taking over the Friedrich Ring shop. His eldest son, Johann Andreas was his successor in Strassburg; his youngest son, Johann Heinrich, a harpsichord and piano maker, was supposedly trained by his uncle Gottfried in Freiberg, before returning to Strassburg to set up a shop. Johann Andreas Stein is known to have worked for Johann Heinrich Silbermann in Strassburg around 1749. The son of an organ builder, Johann Georg Stein, from Heidelsheim in Baden (near Karlsruhe), Johann Andreas Stein became an organbuilder along with his brother Johann Markus, and eventually migrated to Augsburg, where he concentrated on stringed keyboard instruments. Markus was appointed *Hoforgelbauer* in Durlach, then the seat of the court in Baden. Both the Stein brothers probably did work for both Johann Andreas and Johann Heinrich Silbermann in Strassburg during their 'wander' years, and there are no doubt many more unrecorded connections of this kind. Builders certainly moved around extensively from shop to shop as journeyman – that is what 'journeyman' means – and often enough they settled in some available shop, perhaps by marrying the widow of the previous builder. To argue that regional inches would have much, if any, bearing on practical building, save perhaps for some simple cabinet-making, is unrealistic. The assumption that a builder 'always used his local unit of measure' is certainly far from 'clear.'

Historical evidence also contradicts the assumption that a builder can be expected to have always worked with 'convenient numbers and uncomplicated fractions ... Only where it is really necessary and where dictated by some rule or theoretical concept would an instrument builder use a complicated or irrational division of the local unit of measurement.'[32] We have already demonstrated at length in the first part of the article the simultaneous use of both rational and irrational proportions in many historical design traditions, including, in particular, the practice of

[31] Details of the instruments analysed in this study are presented in the Appendix.
[32] O'Brien, 1999, op. cit., p.111.

the Italian Renaissance, which is relevant to the Italian instruments from which the above generalization is extrapolated. By denying the routine use of irrational proportions, it is implied that all builders used modular measurement for their designs, i.e. the possibility of direct geometric construction is excluded. This plainly establishes a bias from the start, contradicting the belief that 'the methodology is impartial and unbiased, being based only on some of the simple geometrical methods and construction principles used by their makers.'[33] As we will demonstrate, a 'different investigator with different preconceptions'[34] can certainly arrive at quite different results, in this case by allowing direct constructive geometry as well as, or instead of, modular measurement.

FINDING THE MODULAR INCH

Regardless of any possible connections with a local unit of measurement, the determination of a builder's modular inch is certainly an important part of the process of understanding the design of an historical instrument. Wraight dismisses the use of local units of measure as an aid to identification, not for the reasons discussed above, but because (to paraphrase with our terminology) he believes it is not feasible to solve the problem of finding a unique modular inch from an extant instrument.[35] His objection is based on the difficulty of distinguishing between local inches from different areas when these are in simple proportional relationships. This may be true if ratios of dimensions are used as the primary basis for analysis,[36] however it is not difficult to obtain a unique modular inch using both absolute dimensions and the geometric relationships between them.

To develop a method for finding the module it will be useful to examine the aspects of an instrument in which it is explicitly exhibited, i.e. to determine how the builder actually used modular dimensions. The initial stages of this analysis must be based only on data within a particular extant instrument itself, obtaining first internal self-consistency and a reasonable implied procedural logic of layout and construction. Further examination of similar instruments by the same builder or school may then be used to support more general conclusions.

Arnaut de Zwolle's instructions for laying out a clavisimbalum (harpsichord) confirm that the module is a variable dimension chosen at the builder's convenience. Arnaut specifies it to be one eighth of the width of the instrument, which the builder could make whatever he 'intended it to be.'[37] The extent to which the maker of a stringed keyboard instrument may have 'exercised his convenience' in choosing his module may be elucidated by considering the design problem at the highest level. Even though the absolute size of a keyboard is somewhat arbitrary, the possible range of dimensions is restricted within reasonable bounds by ergonomic considerations and convention, because it is the physical interface between musician and instrument. For keyboard instruments of any type, the absolute size of the keyboard is therefore a fundamental constraint which must be dealt with in the initial design stages, and this factor has been a central guiding principle from the very earliest instruments. Some sort of direct relationship between keyboard size and module can be expected for any stringed keyboard instrument, just as Arnaut specified in his plans.

For perpendicular instruments,[38] such as virginals or clavichords, keyboard size is the primary physical constraint, and therefore it would have been logical to use this as the starting point for design, probably with a nominal dimension determined by some convention based on the compass. When strings are perpendicular to keys, plucking or strike points will be a function

[33] Ibid., p.155.
[34] Ibid., p.155.
[35] Wraight, op. cit., pp.69-76.
[36] For example: O'Brien, 1999, op. cit. Ratios of orthogonal dimensions at angular positions are used to analyse polygonal virginals and harpsichord tails.
[37] Henri-Arnaut de Zwolle, op. cit. In Arnaut's design the case and keyboard widths are identical because the keyboard projects at the front of the instrument.
[38] Strings and keys at, or close to, a right angle.

of the compass and spacing of the keyboard, and its position in relation to the long side of the box. The box length, i.e. the dimension of the (long) side which contains the keywell, is initially arbitrary. On the other hand, consideration of lateral string spacing implies that box width, i.e. the perpendicular dimension of the case, is a critical design factor related to keyboard compass and spacing, stringband geometry, and mechanical constraints relating to the correct functioning of the key levers. One possibility would have been to begin with a particular box width and from this lay out the box length and keywell position by geometric construction. However this method implies that keyboard size would be dependent on the geometry, instead of being the logically more obvious independent starting point for the instrument. The comparatively small size, and consequently cramped space, of perpendicular instruments means that mechanical functioning of the action becomes that much more critical. For example, there is a minimum required distance from plucking point to the keyfront of a key which will permit the key to function correctly as a double lever. The position and compass of the keywell will determine the subsequent position of the plucking points for top and bottom notes, and this in turn must be related to the string spacing and position of the stringband in the instrument. To accommodate all these constraints implies the diagonal line of plucking points must be carefully located with respect to the holistic geometry of the instrument. The obvious solution is to begin with a keywell of fixed size determined by convention and locate its position on the long side with respect to some reference point on the box (e.g. centre line of the long dimension, right front corner, etc).

These considerations suggest that it would be quite natural for a builder to produce a series of related perpendicular instruments of different sizes, determined by fixing the critical aspects of the geometry and allowing others to vary. The range of Ruckers virginals of different lengths (corresponding to different pitches) illustrates this point. It should be noted that the physical removal of corners of perpendicular instruments, for instance in the construction of polygonal virginals, is irrelevant to the basic rectangular geometry of the instrument. It is very likely that points which define the removed sections were found through simple geometric methods.[39] Due to all these various ad hoc possibilities, establishing a routine method for deriving the modular inch for a perpendicular instrument will be difficult. An important, though somewhat obvious, technique, applicable to builders who made both perpendicular and parallel instruments, would be to avoid the problem, use the extant parallel instruments to determine the modular inch, and extrapolate the use of the same dimension to the extant perpendicular instruments of the same builder. However, it is not clear whether this would always be a valid assumption. The most likely candidate for a generic rule would be some association between keyboard compass and its dimension in modular inches, the same rule probably applying to both the parallel and perpendicular instruments of a particular builder.

For a parallel instrument, the modular inch which was used can be determined with confidence more easily, even though this design is conceptually more complicated than that of a perpendicular instrument. There is a close relationship between the lateral string-spacing and the keyboard width, which is also subject to the same physical constraints as for a perpendicular instrument. Once either of these is fixed, there is little remaining freedom to manipulate the other one. Even so, there is no reason why the two should not have been determined by conceptually independent conventions and the evidence does indicate that this is exactly what was done in historical design practice.

There are several distinct ways in which a builder's modular inch is likely to have been explicitly expressed in the design of an instrument: (i) string spacing (only for parallel instruments); (ii) keyboard dimensions; (iii) structural case width (only for parallel instruments); (iv) long case dimensions and geometry; (v) string lengths; and (vi) short case dimensions. By simultaneously considering, and carefully comparing, data for these six independent design aspects, an accurate and consistent estimate can be determined for the modular inch likely to have been used to construct a particular extant instrument.[40]

[39] O'Brien, 1999, op. cit. This idea is used as the starting point for his analysis.

To support the analysis, dimensional data has been collated for 35 extant harpsichords and pianos of a variety of different types and periods and wide range of keyboard compasses (45 to 82 notes). An estimated value of the builder's *Werkzoll* has been determined for each instrument from the string spacing and used to express reported dimensions as nominal modular inch equivalents. Nominal and observed dimensions are also presented for comparison expressed in mm so that the reader can assess the closeness of the agreement between them.

STRING SPACING

String spacing is the most direct expression of the *Werkzoll*, but only in a parallel instrument.[41] The implied connection with a scale which was presumably used to mark out the action – i.e. keyrack, registers (jack guides), damper rack, and so on – in half modular inch multiples, is often reflected in the builders' terminology for these components. For example, the Talbot manuscript[42] refers to 'over the Jacks proper the Ruler', i.e. the jackrail, a slightly bizarre choice since it not actually marked out at all; the Encyclopédie méthodique[43] describes the action rack as a 'diapason', and the lower jack guide as a 'rule of thin wood'; the concept of 'règle du clavier' is well enough known.[44] Table 1 presents string spacing data for various instruments and an interpretation in terms of an estimate for the implied modular inch used to lay out the stringband. Metrological analysis of these instruments supports the extrapolation that the same modular inch was used by a builder for many aspects of the layout process, including the keyboard.

The most reliable assessment of string spacing can be obtained directly from the original part which was marked out with the ruler, such as the key or jack guide of a harpsichord, or the damper rack of a piano, rather than directly from the stringband itself. To account for any variation in localized spacing, either random or by design, it may be necessary to take measurements on the rack over various ranges of different sizes, avoiding those areas which show any spacing anomaly. It can be expected, and indeed observed on most extant instruments, that string spacing will show considerable consistency and accuracy because it is critical for the correct mechanical functioning of an instrument. Agreement between nominal and observed stringband widths is often within the best accuracy that can be achieved for the observed measurement itself, about 1mm over a full stringband, which converts to an anticipated tolerance of about 0.04mm in the calculated value of the *Werkzoll*. Consideration of nominal half modular inch dimensions on the same instrument, and the inch size expected for the same builder from other instruments, may allow the estimate to be improved further. A physical ruler marked out with spacing according to the proposed inch size is useful for subsequent analysis of the instrument. Sighting this ruler against the string spacing and action rack is also a simple visual means to check the validity of the proposed modular inch because the vernier effect will make

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[40] A modular inch can sometimes be determined by this method to an accuracy as small as 0.01mm, corresponding to an accuracy of 0.1mm on the foot dimension. This reflects the fact that the builder actually did use some unique modular inch value for a given instrument and proper analysis ought to be able to reveal it to the same accuracy as that of the builder's most careful original measurements.

[41] In general, for perpendicular instruments the string spacing is only very indirectly related to the modular inch.

[42] Harpsichord, from the Talbot manuscript. Notes by James Talbot, *c.*1700. Reproduced, in part, in: Hubbard, op. cit. Talbot was an enthusiastic author, but not very knowledgeable about harpsichords, so he may have been confused about exactly which of the cross components were marked in half inch steps.

[43] Diderot and d'Alembert. *Art et métiers méchanique. Instruments de musique et lutherie (art du faiseur d')*, 1785. Excerpts translated are reproduced in: Hubbard, op. cit., p.248 and 251.

[44] Dom Bedos, *The Organ-Builder*, Transl. by Charles Ferguson, Raleigh: Sunbury Press, 1977, Plate LVIII; Dom Bedos' text and diagrams are also reproduced in translation by Hubbard, op.cit., p.222 and Plate XXXVIII). For a discussion of keyboard layout see also: W. Jurgenson. 'The whole truth'. In: 'Matière et Musique - The Cluny Encounter' [J van Immerseel, C Chevallier & T Steiner, editors]. *Proc. of the European Encounter on Instrument Making and Restoration*, Cluny, France, September 1999. Belgium: Alamire, Peer, pp.331-358.)

any consistent deviation quite obvious. The proposed nominal inch size for each of the instruments in Table 1 may include some minimal adjustment of the calculated value based on these considerations.

Table 1. ***estimates of builders' Werkzoll from string spacing analysis***[45]

		stringband measure			*Werkzoll*	
	string choirs[46]	measured spaces[47]	observed mm	nominal inches	calculated mm	nominal mm
C/E-c3						
Ruckers 1637 AR	45	42	582	21	27.71	27.71
C-c3						
Ruckers 1679 IC	49	48	666	24	27.75	27.71
Cristofori 1722	49	48	661	24	27.54	27.56
Cristofori 1726	49	48	661	24	27.54	27.56
C/E-f3						
Ruckers AR1640a	50	49	680	24 1/2	27.76	27.71
C-f3						
Cristofori 1720	54	53	730	26 1/2	27.55	27.56
FF-e3						
G. Silbermann 1749[48]	61	60	801	30	26.70	26.70
FF-f3						
Stein 1783a[49]	61	60	791	30	26.37	26.33
JD Schiedmayer *c.*1794	61	48	633	24	26.38	26.33
Walter pre-1776[50]	61	60	812	30 5/12	26.70	26.70
Walter *c.*1784	61	60	802	30 5/12	26.37	26.33
Späth und Schmahl *c.*1770[51]	62	61	790	30	26.33	26.33
Stein 1783b[52]	62	61	791	30 1/2	25.93	25.93
Attr. Stein *c.*1785	62	61	790	30 1/2	25.90	25.91
Langerer 1793	62	61	815	30 1/2	26.72	26.72
Hofmann 1800	62	61	790	30	26.33	26.33
FF-g3						
Walter *c.*1795	63	62	827	31 5/12	26.32	26.33
Walter *c.*1800a	63	62	827	31 5/12	26.32	26.33
Walter *c.*1800b	63	62	827	31 5/12	26.32	26.33
JL Dulcken *c.*1795	64	62	804	31	25.94	26.00
FF-c4						
JL Dulcken *c.*1798	69	68	886	34	26.06	26.00

[45] See the Appendix for original data and identification information for the instruments in this table.
[46] In piano-building terminology a 'choir' is the set of unison strings used for a given note. Extra spaces which occupy the width of a full string choir in the stringband, e.g. to accommodate gapspacers, have been included in the value cited.
[47] In some cases the measure covers only a portion of the stringband, to avoid obvious areas where spacing is not even.
[48] A transposing instrument. Keyboard compass FF-e3 60 notes. Stringband compass EE-e3 61 notes.
[49] Phase II piano according to terminology in: M Latcham, The Pianos of J.A. Stein, Haags Gemeentesmuseum, 1993.
[50] Walter's idiosyncracy was to allow 73 lines per octave on the stringband, i.e. 6 inches and 1 line. See text for discussion.
[51] See text discussion for explanation of nominal 30 inch stringband.
[52] Phase III piano according to terminology in Latcham, 1993, op. cit.

Streicher 1805	68	67	882	33 1/2	26.33	26.33
FF-f4						
Walter und Sohn *c*.1810	73	72		36		26.33
Streicher 1814	75	74	960	37	25.95	25.94
CC-f4						
Streicher 1808	80	79	1026	39 1/2	25.97	25.93
Dieudonne & Schiedmayer 1815	78	66	870	33	26.36	26.33
Dieudonne & Schiedmayer *c*.1820	79	78	1026	39	26.31	26.33
Graf *c*.1825	79	66	870	33	26.36	26.33
Graf 1826	79	78	1029	39	26.38	26.33
CC-g4						
Fritz *c*.1825	81	80	1053	40	26.33	26.33
Graf *c*.1830	81	80		40		26.33
Boisselot et Fils 1845	84	83	1125	41 1/2	27.11	27.08
Hatzenbuehler *c*.1845[53]	80	79	1071	39 1/2	27.11	27.08
CC-a4						
Erard 1836	86	85	1152	42 1/2	27.11	27.08

Some caution is required with the generalization that rack spacing is always half a modular inch. For certain specific situations this may need modification: (i) *ad hoc* compression or expansion of string spacing over localized areas of the stringband; (ii) idiosyncratic anomalous spacing shifts, for instance as used by Anton Walter in relation to wide spaces allocated for D string choirs; (iii) adjustments for gapspacers in pianos, especially the English style which were usually crammed into a stringband with no extra dumb choir above;[54] (iv) a very large modular inch, which implies some method may have been required to reduce spacing and keyboard width to a manageable size. Some interesting examples of these anomalous situations are described below:

1. Stein 1783a and Stein 1783b differ in that a gapspacer with dumb choir is included in the latter piano.[55] The old 60 spaces (30 old inches) occupy exactly the same dimension (791mm) as the new 61 spaces (30.5 new inches) on the stringband. In order to accommodate the addition of the gapspacer, Stein presumably chose to modify the size of the modular inch, very likely by canting his ruler at 30.5 (old) inches across a space of 30 (old) inches. This method produces a derived inch unit of 25.90mm from the (old) inch of 26.33 which we have determined as that used in the Stein 1783a. The new inch is consistent with all dimensions observed for Stein 1783b and other Phase III Stein pianos. The derivation of this smaller inch in this context seems to have been more widely applied. Within the extant 61 note FF-f3 pianos in the Appendix the larger 26.33 inch is very often observed for those without gapspacers, while the smaller 25.95 inch was used for those pianos with gapspacer and dumb choir (e.g. Attr. Stein *c*.1785, Hofmann 1800). Moreover, Stein's daughter, Nannette

[53] A Parisian builder of Austrian descent. Straight-strung wood-framed upright piano. Stringband width is measured at the strikeline. Spacing of CC-A, i.e. the lowest 10 bass string choirs (those with a single wound string), is compressed at the hitchpin end by ½ inch. Compressed spacing at the hitchpins occupies 4 inches (108mm) for 9 spaces.

[54] A 'dumb choir' is a set of non-sounding strings used to avoid a visible space in the stringband which would otherwise result from the presence of a gapspacer that requires the width of a full string choir to fit it.

[55] This characterizes one of the differences between the so-called Phase II and Phase III pianos of Stein, using the terminology of Latcham, 1993, op. cit.

Streicher, continued to use the smaller derived inch throughout her long career in Vienna where other builders were using the Viennese *Zoll* (26.33mm). The one exception we have observed is the Streicher 1805 FF-c4 piano which is laid out with the larger (old) Stein inch (however this cannot be distinguished from the standard Viennese *Zoll* which is the same size). This may well have been done to provide extra room in the stringband to accommodate the two English-style gapspacers without dumb choirs, which were fitted by shifting the choirs on either side by half a choir width.

2. The Tangentenflügel Späth und Schmahl c.1770 has an FF-f3 stringband at 790mm, an allowance of 30 inches (size 26.33mm), pinned equidistantly with 61 spaces (including gapspacer dumb choir), which can be derived from a canted 30.5 inch over the 30 inch stringband dimension (as in the Stein method described in the previous example). Furthermore, the keytail spacing is narrower still than the stringband/damper spacing, to allow an extra wide space (23+mm) for the damper lifter which passes through the keypanel at the gapspacer. Keytail spacing can be derived separately for left and right half keyboard using a canted 31 inch over the 30 inch stringband dimension: (30/31)*395 = 382mm per half keyboard, giving a nominal central space of 790 – 2*382 = 25mm, consistent with the observed size. This is a variation on Stein's method, and in Späth und Schmahl *c*.1770 three different spacings in effect have been used: 26.33mm (old inches), 25.91mm (string spacing inches), and 25.48mm (keytail inches), although the three units are really expressions of a single (old) modular inch which was used throughout the piano.

3. Anton Walter followed an idiosyncratic practice which reflects both his extended principle of wide D's, which continue to the keytails and string spacing, and the type of gapspacer which has no dumb choir and needs to be fitted in the regular spacing allowance of the stringband. Each octave is pinned with an extra line (i.e. 73 lines, or 6 inches and one line, per octave), and the extra line is added to the space around the D pins, which also accommodates the gapspacer. It is likely that Walter's reasoning centred around the wide D issue, however, since the same practice is followed even in the earlier pianos, which have wide D's on the stringband (the extra one line allowance), but gapspacer between b and c. When calculating the nominal stringband width for a five-octave Walter piano the shift is responsible for an additional 5/12 inch over the stringband width. This is the origin of the unusual nominal dimensions for Walter pianos in Table 1.

4. The Ruckers *c*.1610a AR 2½ *voet* virginal (a unique extant specimen of this size) has a keywell of width 651mm, typical for Ruckers instruments with the C/E-c3 compass. However, keytails with normal half modular inch (27.71mm inch) spacing would require an overall width greater than the longest speaking length, due to the high pitch of this instrument. In order for the instrument to be able to function mechanically the keytail spacing has had to be compressed. This was accomplished by allocating 5½ modular inches per keytail octave, instead of the usual 6 inches, giving a nominal three-octave span of 3 × 5.5 × 27.71 = 457mm, which is in agreement with the observed three-octave spacing on the stringband of this instrument.[56]

In situations where a derived unit has obviously been used to lay out the stringband, it is difficult to distinguish whether the new or old inch has been subsequently used as the *Werkzoll* throughout the instrument. Differences between proposed modular inches are best distinguished on long case dimensions, but any dimension close to (or a multiple of) those which were used in the derivation of the new spacing from the old will not reveal the difference because different sets of nominal dimensions can fit. Those based on either inch can be considered to be consistent with

[56] O'Brien, op.cit, 1990, p.256 and 288-289.

the measurements. In the case of Stein's new inch, however, the fact that Nannette Streicher used the derived string spacing on later and larger pianos with two gapspacers implies that a special ruler had indeed been constructed to the new inch, since the canted method of deriving it would no longer have been applicable for the larger width stringbands on these instruments. Case dimensions on the larger pianos, e.g. Streicher 1808 and Streicher 1814, also tend to agree more closely with nominal dimensions based on the smaller (new) inch, as well. In the absence of such related evidence, for instance for an instrument like Späth und Schmahl *c.*1770, it is difficult to make definitive conclusions.

KEYBOARD DIMENSIONS

Many builders appear to have used a convention based on a multiple of half a modular inch to determine the front keyboard width. This nominal dimension may have been used either for: (i) the space inside which the keys operate –the *keyspace* – i.e. the distance between the inside edges of the keyblocks, or, in a perpendicular instrument, the keywell width; or (ii) the width of the panel from which the keys were cut – the *keysheet*.The first one of these methods seems to have been more commonly used, in which case the keysheet would have been planed narrower than the nominal modular dimension to allow clearance (typically about 2 to 4mm) for it to move between the keyblocks, before dividing it into the appropriate number of natural spaces; the second method implies that the keyblocks would have been installed on the keyframe so as to provide the clearance for the keysheet, giving a keyspace slightly larger than a nominal modular measurement.[57] This proposed simple and pragmatic approach to keyboard layout is well supported by measurement and analysis of extant stringed keyboard instruments (see Table 2).[58]

TABLE 2. ***NOMINAL AND OBSERVED KEYSPACE EXPRESSED IN MODULAR INCHES***

	modular		keyspace or keysheet[59] width			keyspace addition constant	
			nominal	observed		nominal	observed
	inch	notes	inches	inches	mm	inches	
C/E-c3							
Ruckers 1637 AR	27.71	45	23.5	23.46	650	1	0.957
C-c3							
Ruckers 1679 IC	27.71	49	25	25.01	693	0.5	0.509
Cristofori 1722	27.56	49	25	25.01	689	0.5	0.512
Cristofori 1726	27.56	49	25	24.82	684	1	0.319
C/E-f3							
Ruckers 1640b AR	27.71	50	26	26.02	721	1	1.019
C-f3							
Cristofori 1720	27.56	54	27.5	27.53	759	0.5	0.533
FF-e3							
G Silbermann 1749	26.70	60	30	30.00	801	0	0.000

[57] The keysheet widths on Walter pianos consistently agree with this convention.
[58] Three instruments in the Appendix are omitted from this table: Streicher 1808, Fritz *c.*1825 and Graf 1830. The compass of these pianos is one natural wider than similar instruments being built at the same time, and it appears likely the keysheet width resulted from simply adding an extra natural of the size determined by applying a convention to the keyspace of the smaller compass instrument. (See discussion of keyboard natural spacing in this section.)
[59] Italicized values indicate keysheet dimension. All others are keyspace.

FF-f3							
Stein 1783a	26.33	61	31	31.07	818	0.5	0.567
JD Schiedmayer *c.*1794	26.33	61	31	31.07	818	0.5	0.567
Walter pre-1776	26.70	61	31	31.05	829	0.5	0.549
Walter *c.*1784	26.33	61	31	30.95	815	0.5	0.453
Späth und Schmahl *c.*1770	26.33	61	31.5	31.22	822	1	0.719
Stein 1783b	25.93	61	31.5	31.51	817	1	1.008
Attr. Stein *c.*1785	25.91	61	31.5	31.34	812	1	0.839
Langerer 1793	26.72	61	31.5	31.51	842	1	1.012
Hofmann 1800	26.33	61	31.5	31.45	828	1	0.947
FF-g3							
Walter *c.*1795	26.33	63	32	32.02	843	0.5	0.517
Walter *c.*1800a	26.33	63	32	32.05	844	0.5	0.555
Walter *c.*1800b	26.33	63	32	32.02	843	0.5	0.517
JL Dulcken *c.*1795	26.00	63	32	31.96	831	0.5	0.462
FF-c4							
JL Dulcken *c.*1798	26.00	68	35	35.04	911	1	1.038
Streicher 1805	26.33	68	34.5	34.45	907	0.5	0.447
FF-f4							
Walter und Sohn *c.*1810	26.33	73	37.5	37.49	987	1	0.986
Streicher 1814	25.94	73	37.5	37.51	973	1	1.010
CC-f4							
Dieudonne & Schiedmayer 1815	26.33	78	40	39.95	1052	1	0.954
Dieudonne & Schiedmayer *c.*1820	26.33	78	40	39.88	1050	1	0.878
Graf *c.*1825	26.33	78	40	39.95	1052	1	0.954
Graf 1826	26.33	78	40	39.99	1053	1	0.992
CC-g4							
Boisselot et Fils 1845	27.08	80	41	40.99	1110	1	0.990
Hatzenbuehler *c.*1845	27.08	80	40	39.99	1083	0	-0.007
CC-a4							
Erard 1836	27.08	82	42	41.99	1137	1	0.987

As suggested in Table 2 a very simple convention could have been used to determine the nominal keyspace (or keysheet as appropriate) width from the compass: *half the number of notes plus a fixed half inch multiple (keyspace addition constant) gives the nominal keyspace in modular inches.* This pragmatic method of deriving front keyboard size implicitly takes into account the number of natural heads, thus avoiding the necessity of working with the ratio 6/7 by approximating it with a linear relationship in terms of the number of notes and the keyspace addition constant. The addition constant is invariably seen to be a multiple of half a modular inch, with nominal values of 0, ½, or 1 inch being the most common in the examples analysed.

Bigger addition constants are associated with larger keyboard compasses, or compasses such as C/E-c3 with two extreme keys without embedded sharps. For example, the addition constant is one inch for a Ruckers short octave compass (C/E-c3 and C/E-f3) in which both top and bottom natural have no adjacent sharp, and a reduced ½ inch for the chromatic compass (C-c3) in which the bottom note gains an adjacent sharp. Moreover, a large size *Werkzoll* might have suggested to the builder that they compensate with a smaller keyspace addition constant to avoid a very big keyboard octave span.[60] For example, the small Hatzenbuehler upright piano keyboard is reduced in size compared to the keyboards of contemporary French grand pianos with a keyspace addition constant of zero. These considerations do not seem to have influenced the Ruckers, who, despite a rather large modular inch (27.71mm), allocated, for instance, a full 26 inches (one inch addition constant) for their 50 note C/E-f3 keyboards. Keyboards of this same compass on sixteenth-century Venetian virginals may have been given a reduced modular width of either 25 or 24½ Venetian inches (28.98mm),[61] which would imply an addition constant of zero or –½ inch in our terminology. For inches that were much larger than the Venetian inch, it would have been unpractical simply to reduce the keyboard front width in this way. In any case, since significant splaying of keyboards, i.e. wider at the back than the front, is not typically observed in extant Italian instruments (except enharmonic keyboards), this method is unlikely to have been used to manage a very large inch.

A builder could still have used a large inch as the modular unit, but reduced *both* keyboard width and string spacing by canting the modular ruler in the layout procedure. For instance, a reduced-width stringband of say 23 modular inches and keyboard of 23½ inches could be produced this way.[62] As a matter of practicality routine use of this approach to compensate for a large local inch would suggest re-defining the shop inch and adopting the smaller derived value as the new modular inch, just as Stein appears to have done when he needed a smaller inch unit to compensate for the introduction of a gapspacer in the stringband. This is the *Werkzoll* concept of a builder's, or craftsman's, personal inch. Such a modular inch could possibly have been derived from a local inch in some other convenient way, for instance dividing 11 inches into 12 spaces using a canted ruler, to reduce, for instance, a 30mm local inch to a more practicable (11/12)×30 = 27.5mm. In general, no *a priori* relationship between *Werkzoll* and a local inch should be anticipated, though, and it is important to base metrological conclusions only on the evidence presented in an extant instrument, and not on extraneous information or assumptions.

Similarly, a more appropriate builder's *Werkzoll* may have been derived from an inconveniently small local inch. This can be observed, for instance, in a *c.*1715 Keene and Brackley bentside spinet,[63] for which it is reasonably certain that the standard English inch (25.40mm) would have been the local unit. The builders must have considered half-inch keytail spacing too close, because 56 keytails (i.e. 55 spaces centre key to centre key) have been allocated 30 local inches (762mm) instead of the customary 27½ inches. These numbers suggest that Keene used a derived *Werkzoll* which is based on dividing 60 local half inches into 55 new half inches, or, equivalently, he used a modular inch based on dividing the English foot into 11 spaces (27.71mm). At the front of the keys the observed keyspace of 777mm may well have been obtained as 28 modular (derived) inches (27 1/2 + 1/2 inch addition constant). The universality of the English inch makes Keene and Brackley's derivation of the 27.71mm keytail spacing from it obvious enough, however a considerable amount of detailed analysis would be required in order to determine whether this new inch has been used throughout the instrument as the *Werkzoll*, or just for laying out keytails, with the English inch continuing to be used as the *Werkzoll* elsewhere.

A similar procedure may very well have been used by the Ruckers and their contemporaries to arrive at the 27.71mm modular inch. As noted previously association of nominal *voet* specifications with the observed lengths of extant virginals suggests that a foot of 285mm may

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[60] Statistical analysis of this data reveals that the value of the addition constant is positively correlated with both the number of notes in the compass (correl. 0.25) and the number of extreme keys which have no embedded sharp (correl. 0.30), and negatively correlated with the size of the Werkzoll (correl. –0.20).

[61] O'Brien, 1999, op. cit., p.143-145. The implied three-octave spans are 498mm and 508mm respectively, assuming that keysheet width was the nominal half modular inch dimension. If keyspace is the modular dimension the predicted three-octave spans are reduced to about 495 and 505mm, after allowing for clearance.

[62] Compare this to the practice with canted ruler described above for the Tangentenflügel Späth und Schmahl *c.*1770.

[63] Boalch No. 22. Dimensions were obtained from a drawing by John Barnes, Edinburgh, 1984.

have some relevance to the Ruckers' design practice.[64] Division of this foot into 12 inches gives a 23.75mm inch which is the same as the observed front natural spacing on Ruckers keyboards, the space of one octave being allocated 7 of these inches (166.25mm).The Ruckers modular inch (27.71mm) can be derived from this by dividing this same octave span into 6 spaces, as required to lay out the stringspacing for each octave. The corresponding nominal three-octave span is 498.8mm.

As an alternative to this derivation of the modular inch from the front key spacing we should note that the 27.71mm inch is the same size as that obtained by Keene and Brackley by dividing the English foot into 11 inches. The Ruckers modular inch may have had its origins in this same procedure, although we cannot conclude that the generating foot and inch were the English units – these could have been continental measures of the same size. Some connection between the Ruckers and a 12 inch foot of this size has already been noted in our discussion of the comments of Douwes (see footnote 22).

Within a strongly-regulated school of instrument builders, such as the Guild of St Luke in Antwerp, the use of a standard *Werkzoll* could have been enforced by Guild regulation and local statute.[65] A local inch may have been used for this purpose if not of an inappropriate size; for example the modular inch of 27.56mm determined from extant Cristofori pianos is consistent with one of the known Florentine inches of the time.[66] In nineteenth-century Vienna, instrument making was not controlled as a Guild profession, however there was a professional association of keyboard instrument builders (*Bürgerliche Orgel- und Klavierinstrumentenmacher*) with a governing committee of *Kunstgenossen*.[67] The fact that the Viennese *Zoll* (26.33mm) seems to have been almost universally used by Viennese (and many South German) piano builders, even those who were foreigners,[68] points to the adoption of a standard unit of measurement in recognition of the interdependence of the various trades working there. Had the local unit been say 23mm or 32mm, it probably would not have been used by instrument makers without modification and derivation of a more practical unit from it.

The dimensional idiosyncrasies in the pianos of Anton Walter are interesting, but can be quite confusing unless analysed carefully. In addition to his extended wide D principle, which affects string spacing as described above, Walter also used modular inches of two different sizes. The *Werkzoll* used on most extant Walter pianos is identical to the Viennese *Zoll* (26.33mm), however the two earliest extant (61 note) pianos, in Haydn-Haus, Eisenstadt, and Mozart's piano in the Mozart Geburtshaus, Salzburg, are both based on a distinctly larger modular inch of

[64] Connections between *Werkzoll* and local units of measure must, as usual, remain speculative. For the Antwerp region J. Boen, *Maten en gewichten, vergelijkingstafelen van al de oude maten en gewichten met de nieuwe, en omgekeerd... voor elke gemeente der provincie in het bijzonder*, Antwerpen, 1857, gives historical *voet* sizes of 286.8mm (Antwerpen), 278.0mm (Mechelen), 280.4mm (Puurs), 287.43mm (Herentals), and 294.7mm (Mol), and these were typically divided into 11 duimen. D. Gregory, A Treatise of Practical Geometry, 4th Edition. Edinburgh, 1761, gives a value for the (11 duimen) Amsterdam voet of 11.172 (English) inches (283.8mm), a unit that was used in general commerce, ship-building etc. (283.1mm is more commonly cited for the Amsterdam *voet*). The value of 284.6mm for the twelve inch Frankfurt foot given in: C. Bruhns (Ed.), *A New Manual of Logarithms to Seven Places of Decimals*, Tauchnitz, Leipzig, 1870 (English stereotype edition of German original) provides an interesting connection, as circumstantial evidence (O'Brien, 1990, op. cit.) suggests that the Ruckers family may have migrated to Antwerp from that area of Germany.

[65] The Ruckers modular inch of 27.71mm may have been just such a guild standard, as it seems to have been consistently used for a century or more by different builders and shops working in a similar instrument building tradition.

[66] O'Brien, 1999, op. cit., p.168. The Florentine *braccio a terra* is listed as having 20 *soldi* of size 27.56mm.

[67] D. Wythe. 'Conrad Graf (1782-1851) – Imperial Royal Court Fortepiano Maker in Vienna'. Ph D Thesis, New York University, 1990, p.11.

[68] Nannette Streicher is the only exception we have observed.

26.70mm.[69] These two pianos are undated, but they are so similar in many ways that they were probably made at the same time. A date of *c.*1782 has been proposed based on the date when Mozart is known to have acquired his piano, and the assumption that it was new at the time.[70] The difference in modular inch, however, suggests that both of these pianos could have been made prior to the time when Walter settled in Vienna, which is known to be before 1778, and possibly as early as 1774-1776 (when he was aged 22-24).[71] It is, of course, possible that Walter adopted the Viennese inch some time after his arrival, however this is unlikely because ownership of large tools, including measuring and layout tools, was generally forbidden for apprentices and journeymen. That Walter arrived in Vienna as a fully-trained piano builder is consistent with his designation as *Schutzverwandte*, a special classification which allowed foreigners to begin working in their trade upon coming to Vienna. He may well have taken (sample) pianos with him, which reasoning could explain the apparent differences of the Eisenstadt and Mozart pianos, and would suggest that they be given a date prior to 1776 (or even possibly pre-1774). In particular, this metrological reasoning suggests that the piano Mozart acquired from Walter in 1782 may have been at least six years old at the time he bought it.

KEYFRONT OCTAVE SPAN

A design practice based on division of a nominal keysheet width into the required number of natural spaces would imply that keyfront natural spacing (i.e. octave span) was not an explicit *a priori* design decision. Instead, it would have been the implicit consequence of three independent factors, each one of which might have varied separately: (i) the keyboard compass; (ii) the builder's convention for nominal keyspace or keysheet width in modular half-inches; and (iii) the size of the builder's modular inch.[72] This assertion is supported by the data presented in Table 3 which shows the relationship between three-octave span, keyboard compass, and a simple nominal modular dimension for keyspace (or in some cases keysheet) scaled according to the builder's *Werkzoll*.[73] The direct influence of the size of the *Werkzoll* is illustrated by the two FF-f3 compass pianos Walter c.1784 and Walter pre-1776, which both have keysheet widths of 31 modular inches but different absolute dimensions due to variation in the modular inch: 816mm (for 26.33mm inch) vs 829mm (for 26.70mm inch) respectively. This difference is reflected in the respective three-octave spans of 484mm vs 476mm.

The organological value of three-octave span as a *Stichmass* or 'standard measure', i.e. a means to characterize the working practice of a builder, may therefore be questioned. Unfortunately keyboard data is generally collected and is still reported in these terms in the literature, while important dimensions such as keysheet or keyspace, localized variation in three-octave span, and the size of extra-wide top and bottom keys, are not routinely determined. Moreover, these values cannot be calculated from a reported three-octave span with adequate accuracy for dimensional analysis.These observations also support the view that keyspacing was not chosen solely for playing comfort, i.e. expressly chosen for reasons of its own not inherent in the design of the instrument but in the design of the human hand.

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[69] In addition to data obtained directly for the Eisenstadt Walter, this inch size (with Walter's typical 73 line octaves) is consistent with the nut pin spacing reported for both of the pianos in: M. Latcham. 'Mozart and the pianos of Gabriel Anton Walter'. *Early Music* (August, 1997) pp.383-400.

[70] Ibid.

[71] S. Berdux and S. Wittmayer, 'Biographische Notizen zu Anton Walter'. In *Mitteilungen der Internationalen Stiftung Mozarteum*, 48 Jahrgang, Heft 1-4, p.13-106. Salzburg, 2000. Walter's own comments in a legal process in 1796 indicated that he had been in Vienna 'for 20 or 22 years.' His presence in Vienna by 1778 is documented by his purchase of a house.

[72] Henri-Arnaut de Zwolle, op. cit, provides further evidence to support this view. His instructions for laying out a harpsichord generate a variable-sized octave span, according to whatever size is chosen for the module of the instrument.

[73] The extant pianos of Anton Walter show variation in all three of the keyboard parameters, with corresponding variations in three-octave span (see Table 2).

Table 3. *Three-octave span as determined by keyspace, modular inch and keyboard compass*

	three-octave span[75]		nominal keyboard parameters[74]		
			ksp or ksh	*Werkzoll*	nat
	mm	inches	inches	mm	keys
Attr. Stein *c.*1785	471	18.19	31.5	25.91	36
JL Dulcken *c.*1795	472	18.16	32	26.00	37
Streicher 1805	474	18.00	34.5	26.33	40
Streicher 1814	474	18.27	37.5	25.94	43
Stein 1783b	474	18.29	31.5	25.93	36
Stein 1783a	475	18.04	31	26.33	36
JD Schiedmayer *c.*1794	475	18.04	31	26.33	36
Walter *c.*1784	477	18.12	*31*	26.33	36
Späth und Schmahl ca 1770	478	18.15	*31.5*	26.33	36
JL Dulcken *c.*1798	478	18.38	35	26.00	40
Walter *c.*1795	479	18.19	32	26.33	37
Dieudonne & Schiedmayer 1815	479	18.19	40	26.33	46
Dieudonne & Schiedmayer *c.*1820	479	18.19	40	26.33	46
Graf *c.*1825	479	18.19	40	26.33	46
Graf 1826	479	18.19	40	26.33	46
Walter *c.*1800a	480	18.23	*32*	26.33	37
Walter *c.*1800b	480	18.23	*32*	26.33	37
Walter und Sohn ca 1810	480	18.23	37.5	26.33	43
G. Silbermann 1749	482	18.05	*30*	26.70	35
Hofmann 1800	482	18.31	31.5	26.33	36
Hatzenbuehler *c.*1845	484	17.87	40	27.08	47
Walter pre-1776	485	18.16	*31*	26.70	36
Langerer 1793	488	18.24	31.5	26.72	36
Cristofori 1726	491	17.82	25	27.56	29
Boisselot et Fils 1845	494	18.24	41	27.08	47
Cristofori 1720	496	18.00	27.5	27.56	32
Cristofori 1722	497	18.03	25	27.56	29
Erard 1836	497	18.35	42	27.08	48
Ruckers AR1637	499	18.00	23.5	27.71	27
Ruckers IC1679	499	18.00	25	27.71	29
Ruckers AR1640b	499	18.00	26	27.71	30

[74] Keyspace (ksp); keysheet width (ksh); number of natural keys (nat). Where the reported value refers to keysheet width this is indicated by an italicized entry.
[75] Depending on the data source we have derived these values either by: (i) averaging a selection of direct measurements of three-octave span (S) over different areas of the keyboard; or (ii) calculating from observed keysheet width using S = 21*(ksh + ?)/nat, where ? is the space between naturals. It is important, but usually neglected, to include the extra ? space in this fomula, as three-octave span includes the width of 21 naturals and 21 inter-natural spaces, or equivalently the distance between kerf centres when cutting the keys. This extra space typically makes a significant difference of one mm to calculated three-octave spans. When calculating the inverse to get keysheet width from S, the formula ksh = nat*(S/21) – ? should be used. A nominal value of 499mm is given for the Ruckers keyboards.

The most straightforward approach to derive a keyboard inch is simply to divide the keysheet, either trimmed-to-fit in a nominal half modular inch keyspace, or itself a nominal modular half inch dimension, into the number of equal-sized spaces as required for the naturals. Table 2 illustrates how this applies to the instruments in our study. For example, a nominal 32 inch (26.33mm inch) keysheet is typical of Walter's *c.*1800 FF-g3 pianos, and division of this space (843mm) into 37 naturals gives the observed three-octave span of 21×(843/37) + 1 = 479mm (see footnote 75 for explanation of the formula). To illustrate the alternative with nominal modular keyspace, the observed three-octave span of the Cristofori 1722 piano (496mm) is consistent with dividing a keysheet (685mm) trimmed (about 4mm) from a nominal 25 modular inch keyspace (689mm) into 29 equally-sized naturals (21×(689/29) + 1 = 496mm). One can consider how the smaller three-octave span of 491mm was derived for the Cristofori 1726 piano, and why the two extreme keys are wider (about 25mm) than the other keys (23mm). This spacing is consistent with division of a 25 inch keyspace into 29½ spaces, i.e. allowing an extra ½ inch (13mm), which has been distributed between keysheet clearance (2mm), extra width on the top and bottom keys (5mm), and the remainder taken up by setting the keyblocks some 2mm inside the edges of the nominal 25 inch keyspace.

The practical utility of a dedicated keyboard ruler for marking out natural head spacings is obvious, however the origin of the 'keyboard inch' unit,[76] its relation to the builder's modular inch, and the extent to which it was (or was not) used elsewhere in the instrument need careful consideration.[77] An extraneous keyboard inch, i.e. independent of the modular inch, is unlikely, and would not be consistent with the keyspace observations in Table 2, which are clearly strongly related to the modular inch. It is a simple procedure to derive the keyboard inch from a nominal keysheet width by canting the modular inch ruler across it and marking out the required number of naturals. Practicality and the rather acute angle suggest that the two halves of the keyboard might have been marked out separately, with the ruler canted in opposite directions for each half. Alternatively, the operation may well have been done on the benchtop and the derived natural head spacings transferred onto a dedicated keyboard ruler for marking out. A keyboard ruler would have been particularly useful for maintaining a constant three-octave span on similar instruments with different keyboard compasses, for example early nineteenth century Viennese pianos which were made at the same time with at least three different compasses (FF-f4, CC-f4, and CC-g4). In this case, nominal keyspaces cannot all be chosen as half inch multiples, unless the builder is prepared to accept a variable clearance and possibly extra-wide top and bottom keys.

The high production rate of the Ruckers shops, and their practice of gluing fixed width pre-cut natural bone keyplates onto keys after the keys were marked and cut apart, suggest the use of a standard keyboard ruler applied to all compasses. The keyspaces of extant Ruckers instruments, both virginals and harpsichords, are consistently observed (possibly after reconstructing the original specifications) to be very close to nominal sizes of 23.5 modular inches (651mm) for C/E-c3, 25 inches (693mm) for C-c3, and 26 inches (720mm) for C/E-f3.[78] It can be seen that these nominal keyspace sizes are the smallest half modular inch dimensions which will accommodate the required number of naturals at a spacing of 6 modular inches per octave (i.e. 23.75mm per natural head as discussed earlier). Minimum keysheet widths of 641mm (27 naturals), 689mm (29 naturals), and 713mm (30 naturals) respectively are required for the three compasses (with a constant three-octave span of 498.8mm). For example, in 1637AR the

[76] For example see: F. Ernst. 'Four Ruckers harpsichords in Berlin'. *GSJ* XX (1967) pp.63-75.; Heyde, op. cit.; O'Brien, 1990, op. cit.; Koster, 1998, op. cit., p.4.

[77] O'Brien, 1990, op. cit., p.82, suggests that the Ruckers standard keyboard inch may have been derived by allocating 'two Flemish feet' for the width of tweny four naturals. The context suggests that the author is referring to the foot which is determined from the nominal voet lengths of Ruckers virginals, however the keyboard inch would not be compatible with the assumed division of that *voet* into 11 'large' duimen.

[78] This analysis uses dimensions reported in O'Brien, 1990, op. cit. This source contains an extensive datset on Ruckers extant instruments.

minimum C/E-c3 keysheet width of 641mm in a nominal keyspace of 651mm, gives a difference of 10mm which has been allocated to 3mm clearance, and 3mm extra width on the top and bottom keys giving an observed actual keysheet of 646mm. The difference of about 10mm between nominal keyspace and minimum keysheet width is useful for making *ad hoc* adjustments where actual dimensions are not exactly in agreement with nominal ones.

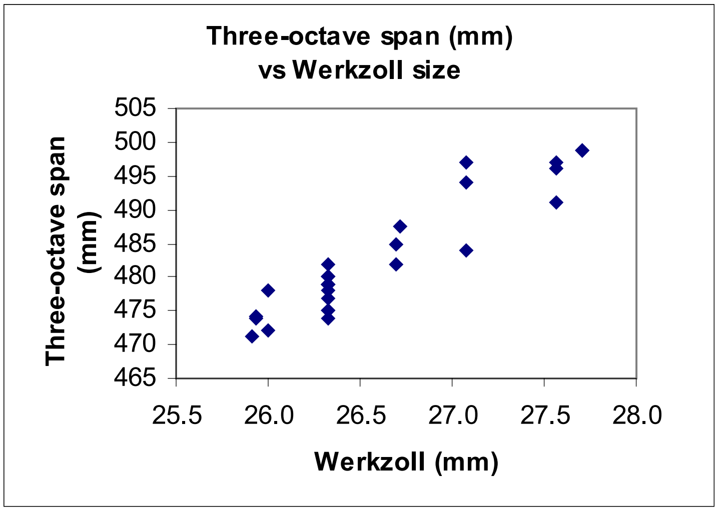
A keyfront three-octave span will naturally be close to 18 modular inches, simply because the keysheet width is similar in size to the corresponding span across the stringband and this spacing is generally based on half inches. It is tempting to suppose that the key-front three-octave span is exactly 18 inches, but this is an over-simplification which does nothing to elucidate the process a builder used to arrive at his front key spacing. The graphs in Figure 2 show three-octave span for the instruments listed in Table 2 as a function of size of the builder's *Werkzoll*, first expressed as an absolute dimension in mm, and second expressed in terms of the *Werkzoll*. There is a clear tendency for absolute three-octave span (mm) to increase with increasing inch size, even though for a given *Werkzoll* a significant range of values results from the various different keyspace conventions and compasses. The second graph reveals a tendency for modular three-octave span (inches) to decrease with increasing size of *Werkzoll*, with a range of values from 17.8 inches to 18.4 inches. Only in a few special cases is the nominal key-front three-octave span close enough to 18.00 inches that it can be considered to be identical at the front and back of the keys: (i) G. Silbermann 1749; (ii) Cristofori 1720 (and probably also Cristofori 1722 is nominally the same), however the Cristofori 1726 is noticeably different; and, (iii) perhaps the most important example, the Ruckers school.

Identical front and back octave spacing over the keyboard cannot be achieved with equal front and back keysheet width.[79] For example, the back of a C/E-c3 Ruckers keysheet is 22 inches between the centres of the extreme keytails, or 22½ inches (622mm) between the outside edges of the keytails. The nominal difference of one modular inch between keyspace (23½ inches) and keysheet width at the keytails (22½ inches) is useful for making *ad hoc* adjustments while laying out the back of the keyboard and the gap spacing. In instruments like the Ruckers the lower guide was installed at the beginning of construction, so there is a practical reason for having some extra room available to mark off at the tails to fit the guide wherever it is, perhaps more to the right or left for a variety of reasons, while at the front, the keyheads remain the same. The Ruckers chromatic C-c3 compass and the short-octave C/E-f3 compass have nominal back keysheet vs keyspace differences of ½ and 1 modular inch respectively.

When front and back octave spacing are different it is generally not difficult to determine which was the starting point and modular inch. The data in Table 2 clearly suggests that the modular inch was invariably the back spacing, as reflected, for example, by its use for nominal keysheet or keyspace widths. The equal front and back spacing of the Ruckers keyboards presents something of an analytical dilemma because practical causality cannot be unequivocally established easily: An equal spacing may have been the result of dividing a width of 24 back spacing inches into 28 spaces, or the keyboard inch may have been the starting point and the back spacing derived from it by dividing 7 keyboard inches into 6 spaces. Distinguishing between these two conceptually different approaches will be an essential part of reconstructing the

[79] The well-known illustration of keyboard layout in Dom Bedos, op.cit. represents a special case that applies to laying out organ keyboards, and should not be extrapolated to stringed keyboard instruments which have different requirements. Dom Bedos' keyboard has identical front and back keysheet width with all keys straight and parallel from front to back. This arrangement is only possible by allowing a non-even tail spacing which Dom Bedos derives by simply extending the divisions marked for the accidentals to the back of the keyboard, i.e. the front and back spacings for naturals and sharps use the same scale. Laying out such an organ keyboard with Dom Bedos' ruler ensures that the multiple stacked keyboards of large organs (as many as five) are identical, with keys lined up perfectly for the correct mechanical operation of pull-downs. Dom Bedos emphasizes the importance of fitting everything very accurately to the scale for this reason (see for instance Article 708, p.153). For stringed keyboard instruments a different spacing front and back is no disadvantage; in fact, it is often required, for instance due to the presence of gapspacers which change the octave relationships between keys and strings.

geometrical methods used by the Ruckers. In this article we have assumed that the Ruckers practice was similar to that which is implied by analysis of the other instruments in the study, and we have adopted a modular inch derived initially from consideration of harpsichord stringspacing. This modular inch is quite consistent with the observed case dimensions of extant Ruckers instruments (e.g. keywell widths), however it should be noted that these are also not inconsistent with use of the keyboard inch as the modular inch.[80] Considerably more research, with examination and measurement of as many individual Ruckers instruments as possible, will be required to clarify the modular inch question.



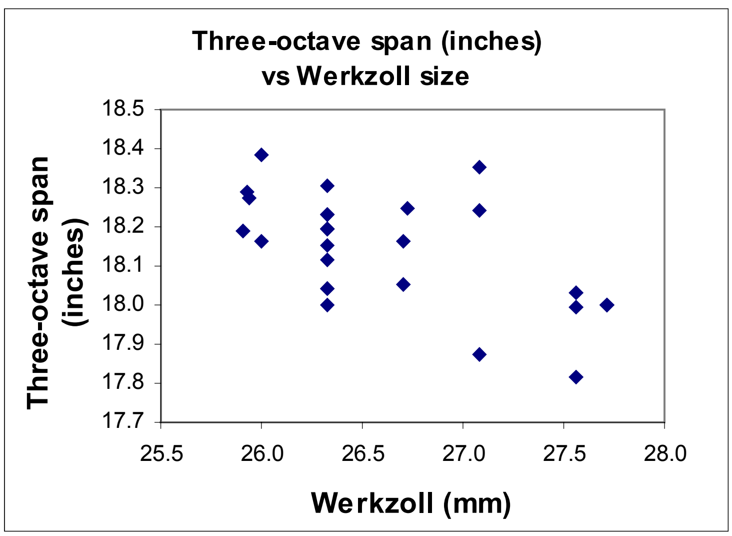


Figure 2. *Graphs of three-octave span expressed in both mm and modular inches vs Werkzoll.*

[80] Koster, 1998, op.cit., p.4, suggests that Ruckers interior widths (i.e. keywell widths) were measured in keyboard inches as the number of naturals plus two. However, this conclusion is based on the average total of 47mm for two keyblocks and clearance cited in O'Brien, 1990, op.cit. Measurement of actual instruments shows significant variation in this dimension, and keywell widths (for instance in the four Ruckers insturments of our study) which are not consistently compatible with Koster's theory. This situation illustrates why the use of average dimensions in these sorts of calculations should be avoided.

STRUCTURAL CASE WIDTH

Consideration of geometric layout methods suggests that some width dimension across the keyboard area of a parallel stringed keyboard instrument is likely to have been used as the module which fixes the absolute size and generates all the other dimensions defining the case geometry and proportions. Moreover, workshop practice suggests that the choice of this generating width dimension must have taken the structural design of the case and the order of assembly into account, simply because it must be known and fixed at the earliest stages of construction. This critical generating dimension will be called the *structural case width*, and denoted by 2W to emphasize the importance which the historical craftsman placed on the centre line of an artefact and the concept of half-width (W).

For example, for late eighteenth- and early nineteenth-century Viennese and South German pianos a logical choice of structural case width would have been the distance across the outside edges of the liners which define the structure of the framing. In most of the designs of these pianos an outer veneered case is glued to the inner framing at a later stage in construction. This thin outer case is functionally a skin which makes no contribution to the case structure, is independent of the case geometry, and serves primarily an aesthetic (or possibly acoustic) function. For most of the closed-bottom designs, the structural case width of this type of piano also corresponds to the width of the bottom boards under the keywell.[81] This concept was illustrated in the first part of this article in which a typical geometric layout procedure applicable to the pianos of J.A.Stein and many other builders was described. For other styles of case framing, the builder's choice of structural width dimension can generally be determined with confidence by consideration of the construction sequence. For instance, for a Ruckers harpsichord or a mid-nineteenth-century piano in which the rim and outer case are the same, it would logically be the overall outside width that was used.

As the structural case width is the generating dimension for an instrument there is some degree of latitude in its size. Consequently, there would have been no reason not to have fixed a nominal value as some convenient modular half-inch multiple. The main constraint on the size of 2W is that the keywell space it defines must be large enough to accommodate the action and key assembly. In particular, the choice of 2W must allow sufficient room for the keyspace, the keyblocks, clearance for the action frame and other keywell hardware, keyboard shift for transposition or una corda stop, and the thicknesses of the bass and treble keywell structural framing members (liners). Furthermore, we can expect in general that thicker liners will be associated with instruments with larger compass keyboards simply because these have more strings and required heavier framing. Bigger instruments are also likely to be later instruments and will probably require extra clearance space in the keywell for keyboard shift and possibly pedal trapwork. Table 4 gives observed and nominal values for 2W expressed in terms of the builder's *Werkzoll* appropriate for each instrument.

In selecting this generating width dimension for an instrument, builders may have followed a common general principle similar to that we have suggested for the keyspace dimension. A simple linear relationship with addition constant to determine 2W from keyspace (ks) or the number of notes (N) is not likely, because all three of these increase at the same rate with increasing compass and the required increase in the non-keyspace width would have to be obtained by changing the addition constant. This problem is avoided by generating 2W from the number of natural keys (nat), which is typically about 60% of the total number of notes on any keyboard.[82] Therefore, with increasing compass, the number of natural keys increases at a rate of about 60% that of N, while keyspace increases at the same rate as N/2. Consequently a convention for 2W based on the number of naturals would allocate about 10% of the structural case width to non-keyspace purposes, a value which is consistent with the observations of extant instruments. In fact, as shown in the graph in Figure 3, the best-fit relationship between 2W and

[81] The J. L.Dulcken *c.*1795 piano, in which the outer case rests on the bottom, is an exception to this.

[82] The ratio number of natural keys to N varies between about 58% and 60% for the keyboard compasses of the instruments in the Appendix.

the number of natural keys (assuming no intercept) over the full set of instruments in Table 4 is simply that 2W (modular inches) = the number of natural keys. Actual values of 2W for individual instruments reflect the functional characteristics of a particular design in terms of the allocation of the non-keyspace part of the width. The observed values of 2W for the instruments in this study can be obtained consistently and accurately from the number of natural keys plus a small multiple of half a modular inch (*width addition constant*) varying between –1½ and +1 inch.

TABLE 4. *RELATIONSHIP BETWEEN STRUCTURAL CASE WIDTH AND KEYBOARD DATA*

				structural width 2W		addition constant	width allocation		
							keysp[83]	non-keysp	
	Werkzoll			actual	nominal	2W– nat	ks	2W – ks	
	mm	notes	nats	inches	inches	inches	inches	inches	%2W
C/E-c3									
Ruckers 1640a AR	27.71	45	27	26.38	26.5	-0.5	23.5	3	11.3
Ruckers 1637 AR	27.71	45	27	25.91	26	-1.0	23.5	2.5	9.6
C-c3									
Ruckers 1679 IC	27.71	49	29	27.43	27.5	-1.5	25	2.5	9.1
Cristofori 1722	27.56	49	29	28.01	28	-1.0	25	3	10.7
Cristofori 1726	27.56	49	29	28.23	28.25	-0.8	25	3.25	11.5
C/E-f3									
Ruckers 1640b AR	27.71	50	30	28.87	29	-1.0	26	3	10.3
C-f3									
Cristofori 1720	27.56	54	32	32.58	32.5	0.5	27.5	5	15.4
FF-e3									
G. Silbermann 1749	26.70	61	35	36.03	36	1.0	30	6	16.7
FF-f3									
Stein 1783a	26.33	61	36	35.51	35.5	-0.5	31	4.5	12.7
JD Schiedmayer c.1794	26.33	61	36	35.70	35.5	-0.5	31	4.5	12.7
Walter pre-1776	26.70	61	36	35.92	36	0.0	31	5	13.9
Walter c.1784	26.33	61	36	36.08	36	0.0	31	5	13.9
Späth und Schmahl c.1770	26.33	61	36	35.93	36	0.0	31.5	4.5	12.5
Stein 1783b	25.93	61	36	35.94	36	0.0	31.5	4.5	12.5
Attr. Stein *c.*1785	25.91	61	36	35.62	35.5	-0.5	31.5	4	11.3
Langerer 1793	26.72	61	36	35.52	35.5	-0.5	31.5	4	11.3
Hofmann 1800	26.33	61	36	35.55	35.5	-0.5	31.5	4	11.3
FF-g3									
Walter *c.*1795	26.33	63	37	36.54	36.5	-0.5	32	4.5	12.3
Walter *c.*1800a	26.33	63	37	36.99	37	0.0	32	5	13.5
Walter *c.*1800b	26.33	63	37	36.88	37	0.0	32	5	13.5
JL Dulcken c.1795[84]	26.00	63	37	37.00	37	0.0	32	5	13.5
FF-c4									

[83] Nominal half-inch value, either keysheet width or keyspace according to observation.
[84] The outer case rests on the bottom in this piano design. Structural width is taken to be the distance across the outside of the liners, following the analogy with the more common Viennese piano design in which the outer case is glued to the liner overlapping the bottom. The geometry of the J.L.Dulcken 1795 implies this choice was indeed the builder's generating dimension and was presumably marked on an over-size bottom when laying out and constructing the inner framing.

Streicher 1805	26.33	68	40	38.66	38.5	-1.5	34.5	4	10.4
FF-f4									
Walter und Sohn *c*.1810	26.33	73	43	42.46	42.5	-0.5	37.5	5	11.8
Streicher 1814	25.94	73	43	43.87	44	1.0	37.5	6.5	14.8
CC-f4									
Streicher 1808	25.93	78	46	45.47	45.5	-0.5	40.25	5.25	11.5
Dieudonne & Schiedmayer *c*.1820	26.33	78	46	44.78	44.75	-1.3	40	4.75	10.6
Graf 1826	26.33	78	46	44.66	44.5	-1.5	40	4.5	10.1
CC-g4									
Fritz *c*.1825	26.33	80	47	46.60	46.5	-0.5	40.75	5.75	12.4
Graf *c*.1830	26.33	80	47	45.88	46	-1.0	41	5	10.9
Boisselot et Fils 1845	27.08	80	47	43.94	44	-3.0	41	3	6.8
CC-a4									
Erard 1836	27.08	82	48	46.90	47	-1.0	42	5	10.6

Discovering how the nominal 2W value was derived is not the important thing here. In fact, the actual convention undoubtedly varied between builders and, in any case, a proposed method must remain speculative in the absence of specific recorded evidence. The key fact which can be established with reasonable certainty by analysing these extant instruments is *that the nominal dimensions used for case width were invariably a multiple of half a modular inch.*[85] This implies that it was indeed this dimension which was the explicit design measurement. A builder may have solved his design problem by simply choosing the closest modular half inch dimension for 2W which allocated enough of the width to non-keyspace purposes to fit the projected design of the instrument. Smaller fractions of an inch would not have been needed to define a nominal 2W, because an additional ¼ inch on the width has little implication for the available space, adding a negligible 3mm to the clearance on each side if all other factors remain the same.

The last column in Table 4 shows the ratio of the non-keyspace part (2W – ks) as a percent of case width (2W). The relationship between this ratio and the width addition constant (2W – nat) is graphed in Figure 4. The absolute size of the non-keyspace part of the width is also shown as a function of the addition constant. Relatively thin walls and the absence of separate keycheeks on the Ruckers harpsichords allowed a negative addition constant of –1 or –1½ inches with only about 9-10% of the width allocated to non-keyspace use. Five-octave Viennese pianos required a larger addition constant, typically zero, with about 13% of the width for non-keyspace use. A negative addition constant between –½ and – 1½ inches is seen on the larger pianos with non-keyspace width ratio of about 10-12%. Positive addition constants are observed on only a few of the instruments. For example, the 6 octave Streicher 1814 piano which has thick keywell veneer lining, internal pedal trapwork, and an una corda shift, allocates about 15% of the case width to non-keyspace use, and has an addition constant of a full 1 inch. The Silbermann 1749 piano shows the largest width allocation to non-keyspace use at 17% with an addition constant of 1 inch, presumably a reflection of the fact that the piano is a transposer.

The extant pianos of Conrad Graf provide a source of data for a large number of instruments from a relatively stable building tradition over a period of about 20 years. For the most part these pianos were undated, but signed with Opus numbers, so it is possible to establish a chronology. Table 5 shows the results of analysing case width including the outer case and veneers, but excluding the mouldings.[86] Unfortunately the widths of the bottom boards were not reported, so some caution must be exercised in interpreting the values in the context of structural case width.

[85] The difference in observed and nominal means (sample size 29) is 0.15mm (95% Confidence Interval: -0.86mm to 1.16mm). This provides strong evidence that the observed and nominal means are within 1mm.

[86] This analysis uses dimensions reported in Wythe, op. cit.

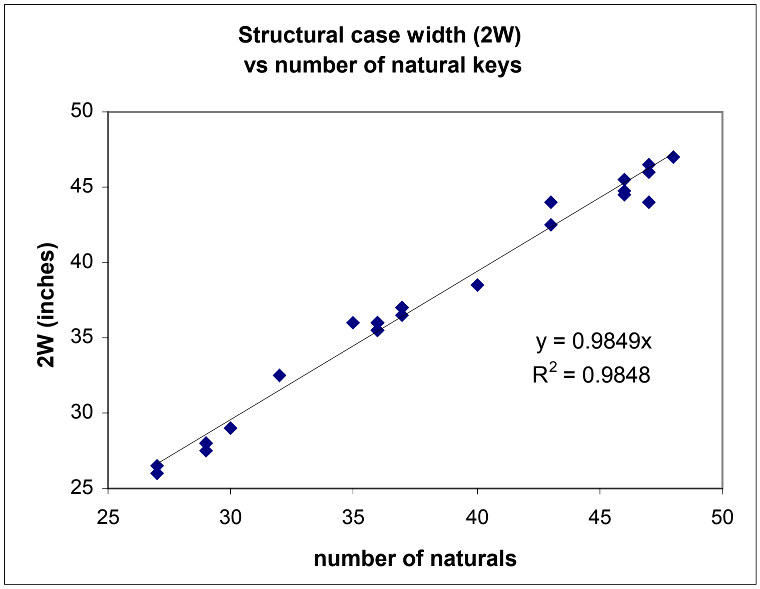


Figure 3. *Graph of structural case width (2W) vs number of natural keys*

Nevertheless, it is possible to estimate 2W from the data based on the observation that, on Graf pianos, the two outer case walls including (nominally ½ inch) cores and (nominally 2 line thick veneers) generally contribute to the outer case width about a nominal one Viennese *Zoll* and four lines. In practice, values between about 32 to 39mm are observed. After collating the data it becomes clear that Graf used different nominal case widths even within the output of pianos of the same compass, and that these different widths occurred in groupings. In addition to the small number of FF-f4 pianos, two distinct groups of CC-f4 pianos, and three of CC-g4 can be identified based on the outer case width. This analysis suggests that Graf changed his nominal case width for CC-f4 pianos somewhere between Opus 654 and Opus 693, and changed it for the CC-g4 pianos twice, between Opus 1490 and 1594, and between Opus 2716 and 2730. The results of statistical analysis of this data are presented in Table 5. Mean outer case widths have been used to estimate a possible nominal value for 2W as a multiple of half a modular inch for each group of pianos. The outer case wall thicknesses implied by these estimates (between 31 and 39mm) are consistent with observed typical outer case wall thicknesses for Graf pianos. The estimated nominal structural case widths are also consistent with the observed values for the CC-f4 Graf 1826 (Opus 995) and CC-g4 Graf *c.*1830 (Opus 2318) pianos in the dataset of the Appendix. It is interesting that the first of the changes in nominal structural case width for CC-g4 pianos corresponds with the adoption of an open bottom framing, presumably because Graf felt it desirable to increase the thickness of the liners by ¼ inch, requiring an extra half inch on the 2W dimension. In conclusion, dimensional analysis of these 50 extant Graf pianos provides support for a nominal 2W value having been selected by the builder as a multiple of half a modular inch.

Structural case width is a logical and practical generating dimension to use in laying out a parallel stringed keyboard instrument. Marking out a single pre-determined conventional structural width on a bottom template is much easier and more reliable than fussing with adding separate allowances for keywell space (possibly having to worry about lining veneers as well), framing thicknesses and clearance. For instruments not built on the bottom, such as Ruckers harpsichords, the wrestplank can be marked at the pre-determined conventional width to show the outside of the case, or, perhaps simpler still, the bellyrail (which fits into mortises on the cheek and spine) may be measured quickly and cut say at a length 1/2 inch less than required 2W. Standard bellyrails at various nominal lengths for different instrument designs could have been prepared and kept in a bin as stock parts in a busy shop such as the Ruckers. In all cases, for a given keyboard compass the desired nominal width for a particular instrument design would only have had to be determined once and the dimension recalled by some simple convention thereafter.

Table 5. *Statistical analysis of case width of 50 Graf pianos*

		Sample		Outer case width Mean (standard error)		Nominal estim. 2W	Implied outer case	
Compass	**Opus**	**size**	**nats**	**mm**	**inches**	**inches**	**inches**	**mm**
FF-f4		6	43	1147 (4)	43.54 (0.17)	42.5	1.04	27
CC-f4	365-654	6	46	1229 (1)	46.68 (0.04)	45.5	1.18	31
CC-f4	693-1245	11	46	1211 (1)	45.99 (0.04)	44.5	1.49	39
CC-g4	1018-1490	6	47	1233 (2)	46.84 (0.06)	45.5	1.34	35
CC-g4	1594-2716	17	47	1248 (1)	47.39 (0.05)	46	1.39	37
CC-g4	2730-2788	4	47	1268 (3)	48.17 (0.10)	47	1.17	31

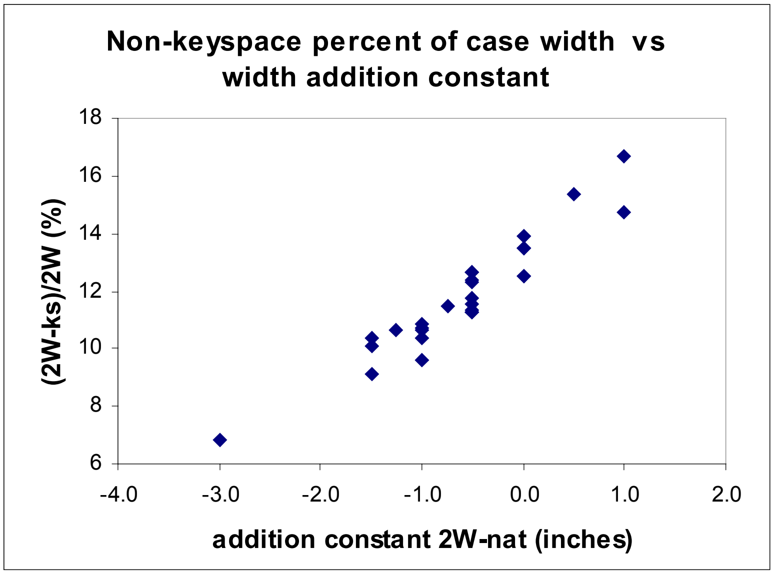
LONG CASE DIMENSIONS AND GEOMETRY

The use of direct geometric construction, simple modular measurement, or some combination of these is often not immediately obvious in the design of an extant instrument. It is therefore important in the initial stages of analysis to avoid restrictive *a priori* assumptions and give due consideration to all potentially relevant geometric possibilities. For example, the assumption that the angles of cutoff sections on Italian virginals and tail angle of harpsichords were defined by measuring orthogonal dimensions[87] ignores the possibility that direct geometric construction with the trammel could have been used. Even though many, if not all, of the essentially arbitrary long case dimensions may well have been derived as simple modular measurements, dimensions constructed geometrically will generally have no direct modular relationship. A complete analysis must consider both possibilities for a particular instrument. A proper historical perspective and degree of common sense are essential. Some confidence can be derived, for instance, from results which imply that a builder may have used ratios of simple whole number dimensions such as 8:10 or 2:6 modular inches. However, it is highly unlikely that ratios of implied dimensions such as 7½ to 2 or 12¾ to 10½ modular inches[88] would have been used, indicating that either the associated cutoff angles were a consequence of some other method than orthogonal measurement, or that the proposed modular inch is incorrect, or both. Applying a restrictive theory to an extant instrument may produce numerically correct, but pragmatically highly unlikely, results, and consequently false conclusions about the builder's working practice.

To establish the likelihood that direct constructive geometry was used requires very accurate data on all relevant case dimensions. This is especially true of the generating dimensions for the geometry – keyspace for a perpendicular instrument and structural case width for a parallel instrument – which would have been marked out initially by the builder and from which all subsequent dimensions are explicitly constructed. Any initial inaccuracy in the size of 2W, for example, will be reflected proportionally in the dimensions which are generated from it. An holistic view of the overall design may indicate that alternative geometric constructions, with different associated modular inches, may be possible, and analysis should be directed at distinguishing between the various possibilities. To illustrate how easily this situation arises, we have analysed the geometry of a 1552 polygonal virginal by Franciscus Patavinus and devised two alternative solutions consistent with the reported baseboard dimensions. For the purpose of comparison, the geometric scheme proposed by the original author,[89] based on the Venetian inch (28.98mm), has also been reconstructed in the diagram. The original solution has a considerable

[87] O'Brien, 1999, op. cit., p.112. It is also stated (p. 154) that the lengths of the sides of rectangular virginals were usually measured in whole number multiples of the local unit of measurement, ignoring other possible geometric constructions..

[88] Ibid., p.136 and 138. These are the actual values which are proposed as arbitrary dimensions used by the Italian builder Bolcioni in his design.



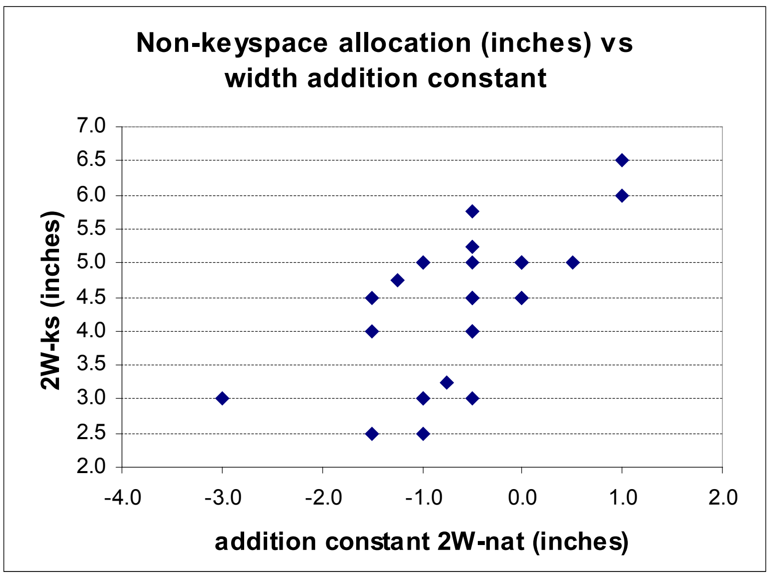


Figure 4. *Non-keyspace width vs addition constant.*

number of arbitrary fractional dimensions required to be laid out by the builder, suggesting that some aspects of the design may actually have been derived by geometric constructions with a trammel, rather than by modular measurement. Putting aside the modular ruler, a simple proportional analysis reveals two important facts. First, the right side of the instrument is a 1:2 rectangle of sides 362 and 724mm; and, second, the diagonal of this 1:2 rectangle, 362√5mm, is exactly half the length of the instrument. Allowing a combination of modular rule and trammel, it is simple enough to devise the two alternative constructions for this Patavinus virginal, which are as consistent with the bottom dimensions as the previously published solution, and require no

[89] Ibid., pp.113-21. Baseboard dimensions used in our analysis and figures are those reported in the text data.

complicated fractional modular dimensions. All three of the implied modular inches are quite different:

Method 1. Figure 5b. Modular inch = 30.17mm

Over-length bottom boards planed to a width of 12 inches and squared at the right edge AB; points C and E marked on the back edge at 14 and 24 inches; arc AC swung down with the compass to the front edge to define point D, the right edge of the right keywell brace; from D the right edge of the left keywell brace (F) marked at 24 inches and the left front corner of the instrument a further 11 inches (G); arc GH, defined by the midpoint of side GI, swung down to define point J at distance 2 inches inside the left edge.

Method 2. Figure 5c. Modular inch = 27.85mm

Over-length bottom boards planed to a width of 13 inches and squared at the right edge; 1:2 rectangle marked from the right edge by measuring point C at 26 inches; diagonal AC swung down with the trammel to the front edge to define midpoint D; arc DA centred at D swung to define left front corner of the instrument (E) – overall length is 26√5 inches, or 1619mm; right edge of right keywell brace (F) marked at 20 inches from A; right edge of left keywell brace (G) marked at 26 inches from F; point I located on perpendicular from G; point K defined by intersection of arcs IJ and EH, where J is the back left corner and H is the midpoint of side EJ; L located by swinging arc AF to meet the back edge.

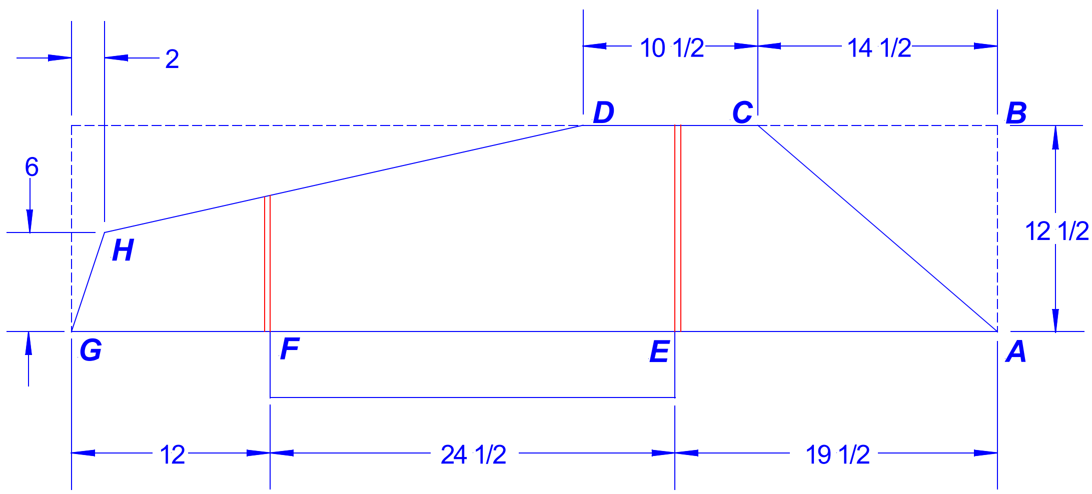


Figure 5. *(a) Patavinus 1552 Virginal. Reconstruction of O'Brien's geometric solution allowing only modular measurement. Modular inch = 28.98mm.*

Method 1 involves arbitrary modular dimensions in only simple whole numbers of 11, 14, 12 and 24 inches;[90] Method 2 involves trivial modular dimensions in whole numbers 13, 20 and 26 inches, numbers which are quite characteristic of modular dimensions often used by historical craftsmen. These constructions are very easy to memorize and implement, involving no fractions of the modular inch, and either one of them predicts nominal measurements which are in excellent agreement with the reported ones.

This example illustrates the difficulty of distinguishing between irrational dimensions, or ratios of dimensions, which are the result of transferring diagonal measurements with a trammel or compasses to orthogonal positions within an instrument, and rational dimensions, which can be

[90] The first two dimensions are reminiscent of {14, 11, 3} dimensions, which can be traced back as far as Arnaut.

associated with direct measurement from a modular scale, typically restricted to whole or half inch multiples. For instance, it is easy enough to convince oneself that a builder marked out a dimension inaccurately, intending it to be twice as long as some other dimension, when, actually, the dimension was a highly accurate √5 length derived from the diagonal of a 1:2 rectangle. Rational approximations of irrational dimensions are even more difficult to distinguish. For example, the Fibonacci numbers provide the ratio 8/13 which is extremely close to the golden ratio[91] (ϕ), and the well-known {14, 11, 3} dimensions provide the rational fractions 14/3 and 11/14 which approximate $2\sqrt{5}$ and $\sqrt{\phi}$ respectively. Faced with the dimensions of an extant instrument, careful analysis is required to determine whether ratios such as these were obtained as rational approximations using modular measurement, or as irrational ratios using a direct geometric construction. This situation is ameliorated to some extent by considering the accuracy of the relationships – dimensions that were constructed with a trammel can often be identified by virtue of their extreme accuracy and self-consistency within a particular instrument.

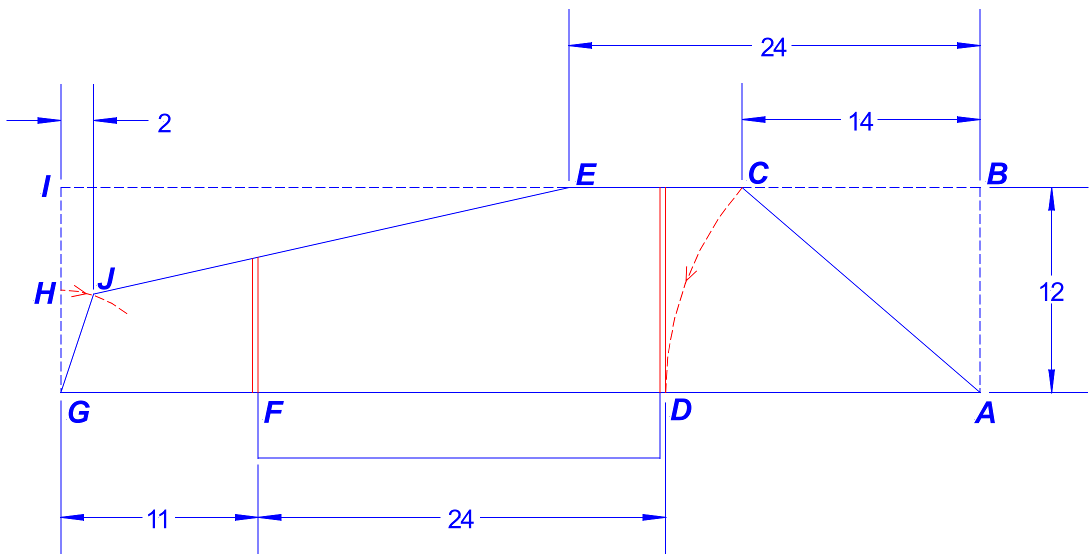


Figure 5. *(b) Patavinus 1552 Virginal. Alternative construction method 1. Modular measurement and trammel construction. Modular inch = 30.17mm.*

Referring to his examination of many Italian virginals and harpsichords, O'Brien comments that he has 'found no evidence whatsoever … that the makers were in any way concerned with the use of the Golden Ratio.'[92] In fact, his first example, the Patavinus virginal, demonstrates strong connections to ϕ, as evident in the √5 geometry derivable from the diagonal of the obvious 1:2 rectangle, and the ratio between case length and width at $2\sqrt{5}$. Furthermore, the left corner angle of the Patavinus virginal of 72° is an angle which should immediately be recognizable as the characteristic base angle of a golden triangle (Figure 6a), an isosceles triangle with base in the ratio of ϕ to 1 with the sides. Using the alternative possible modular inch of 27.85mm (from Method 2 above) the case side is 13 inches. Setting a compass on this edge, swinging an arc centred at the front left corner, and marking a point where the arc is 4 modular inches from the left edge, will construct half of a golden triangle, and define the angle of the cutoff line at the observed 72° (Figure 6b). This trivial construction for the left corner of the Patavinus virginal

[91] See Part I for a discussion of the golden ratio (√5 – 1) / 2.
[92] O'Brien, 1999, op. cit., p.157.

using the rational approximation 8/13 for the golden ratio, is typical of the historical builder's pragmatic use of the golden ratio and √5 geometry, and marked-out dimensions based on Fibonacci numbers.

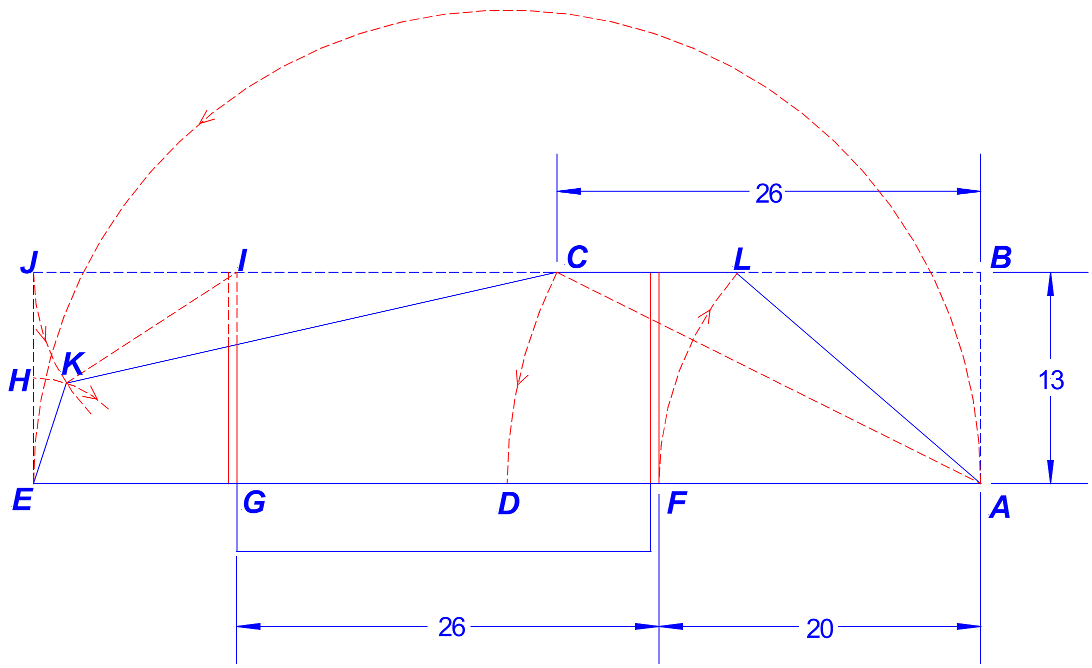


Figure 5. *(c) Patavinus 1552 Virginal. Alternative construction method 2. Trammel construction with minimal modular measurement. Modular inch = 27.85mm*

STRING LENGTHS

String lengths consistent with nominal modular inch dimensions may be useful evidence in support of a proposed modular inch size determined from string spacing and (or) case and keyboard geometry. However, this can never be definitive, because, even if nominal lengths are known, they may not have been implemented exactly, due to practical ad hoc adjustments at the various stages of construction.

Historical sources indicate that the string scaling practice used by builders of harpsichords and virginals was based on simple nominal string lengths. This method is frequently described with reference lengths generally associated with C strings.[93] If desired, quite sophisticated non-Pythagorean scaling may easily be achieved with nominal modular string length dimensions, either by including an addition constant for each length, or by shifting the reference note to a different scale location. Such simple methods with nominal modular string lengths were still being described by piano builders as late as 1886.[94]

The scaling traditions used by some piano builders are perhaps more sophisticated and less clear than those of harpsichord and virginal builders. An historical source describes a

[93] Such tables are well known. For example: Douwes, op. cit. His tables of string lengths are reproduced in: Hubbard, op. cit., p.233-4.
[94] J. Blüthner and H. Gretschel. *Lehrbuch des Pianofortebaues in seiner Geschichte, Theorie und Technik*. Weimar, 1886 (second edition), p.101.

proportional geometric method for constructing string lengths for a piano scale.[95] This relies on monochord tests of desired string gauges to determine lengths consistent with the available wire strength. String lengths in a scale designed in this way are expressed in arbitrary units and may be transferred to an instrument without converting them to nominal modular dimensions.

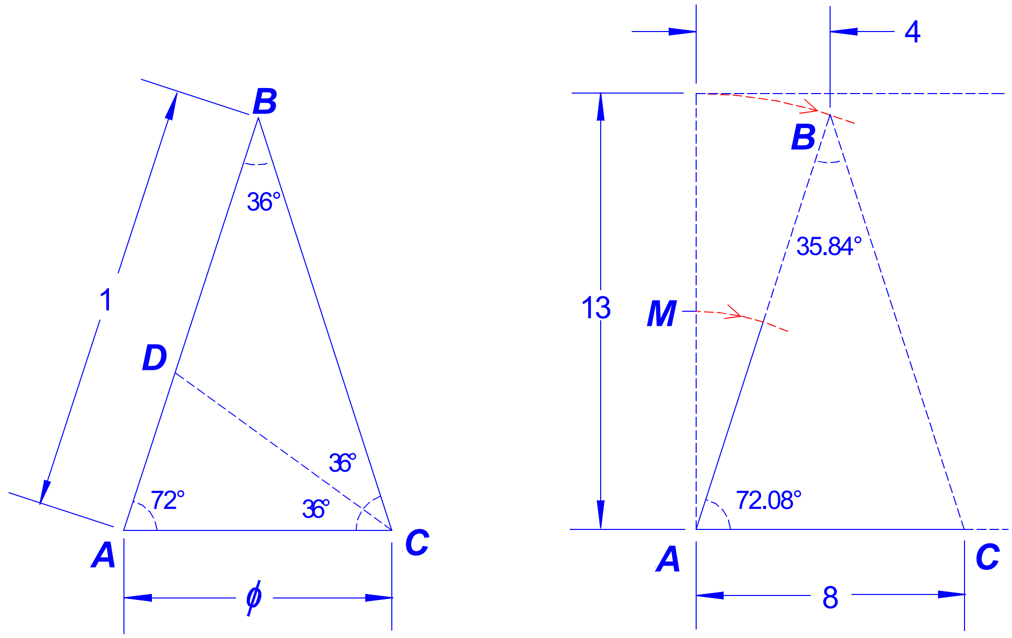


Figure 6. *(a) (left) Golden triangle ABC. AC:DB = AC:AB is the golden ratio;(b) (right) Application of golden triangle to left corner angle of Patavinus 1552 Virginal. Modular inch = 27.85mm. Rational approximation of golden triangle with base to side ratio 8:13 and location of cutoff point from case side midpoint M.*

String lengths in historical keyboard instruments, including pianos, show significant variation, both within and between instruments of the same type from a particular builder. It would be presumptuous to suggest that this variation represents 'errors' or 'inaccuracies', as it is quite possible that an historical builder may not have thought so precisely about string lengths as many modern organologists suggest they did (or should have). Nominal c2 lengths of say 12 vs 13 modular inches may be easily distinguished in extant instruments, however smaller variation of say half an inch on the c2 length represents only about two thirds of a semitone and does not necessarily imply a different pitch was intended by the builder. Even the assumption that string lengths themselves were explicitly measured in practice may be questioned. It is not inconceivable, for instance, that the location of one or other edge of the bridge was determined by geometry, or modular measurement, perhaps locating the front edge of the bridge with respect to the back edge of the nut, or in relation to the edge of the case. In this case string lengths would simply have been the implicit consequence of the bridge width and local angle after gluing it in position. Apparent bridge and string locations marked on the bottom boards, as observed for instance with some Italian harpsichords, should also be interpreted with caution. These are often not in agreement with the actual final location of the bridge, and may have served only as a temporary reference for constructing the bentside curve on the bottom. In order to use string

[95] 'Historische Beschreibung der aufrechtstehenden Forte-Pianos, von der Erfindung Wachtl und Bleyers in Wien', *Intelligenz-Blatt zur aligemeinen musikalischen Zeitung*, No. XVII. Leipzig, November 1811. This is the method described by the piano builder Bleyer.

length data meaningfully, an *a priori* nominal scheme must be known or obvious, since every scale contains a string length which could conceivably be described as 'approximately 12 or 13 modular inches'. In this exercise again, common sense and historical perspective play an important role.

SMALL CASE DIMENSIONS

Small case dimensions less than about 12 modular inches, for example the thicknesses, widths, or heights of components such as braces, bottoms, case sides and so on, can have some limited utility in support of a proposed modular inch. Over the range of typical modular inch sizes used by builders (25.91mm to 27.71mm for the instruments in the Appendix), differences can still be significant even for quite small dimensions of the same nominal size. At the other extreme, in order to distinguish say a new 25.91mm derived modular inch from the old 26.33mm Viennese *Zoll* on a component dimension like the width of the wrestplank, identification of a difference of about 3mm is required, which is indistinguishable from random variation. It is also possible that a builder may have used available standard local timber thickness for some case components, and, of course, these would not be related to the *Werkzoll* of the instrument. Some of the small dimensions, such as the thickness of a wrestplank, were more critical and consequently may have been marked out with more precision. Therefore it may still be possible to observe meaningful differences in these specific small dimensions for different sizes of modular inch.

Even if it is not possible to distinguish a specific builder's modular inch in small case dimensions, the nominal values used for these components are themselves of considerable independent interest. To illustrate how modular measurement pervaded the design of an instrument, wrestplank widths (excluding any protruding top veneers) measured from front to back are shown in Table 6 for some of the instruments in the Appendix. The agreement between the observed values and nominal half modular inch (occasionally ¼ inch) dimensions is very close, as shown in the graph of Figure 7. The values for instruments with skewed gap line, such as Walter and Graf, suggest that the specific angles were obtained very simply by using different modular widths at the bass and treble ends of the wrestplank. The origin of Walter's characteristic 89° gap angle is trivial in the context of modular measurement and a wrestplank that is ½ inch narrower at the treble end than the bass end.[96]

CONCLUSIONS – FINDING THE MODULAR INCH

In order to derive with confidence an accurate value for the size of the modular inch which was used in the construction of an extant stringed keyboard instrument it is necessary to synthesize observations from various independent aspects of the design. A pragmatic approach is required to find the most likely value that best fits the evidence. The following aspects of instrument design may reflect the builder's use of modular measurement, in terms of dimensions that are based on multiples of half a modular inch:

1. **String spacing for parallel instruments.** This is likely to have been the most strictly accurate use of the modular scale in an instrument as it is critical for correct mechanical functioning. Some care is required with analysis to ensure that the impacts of localized random variation, or a builder's intentional irregularities in spacing, are properly taken into account in calculating the implied modular inch.
2. **Keyboard dimensions.** For the instruments in this study the evidence indicates that the front keyspacing was generated from a nominal half modular inch dimension. This may have been applied to the keysheet width itself or to the keyspace. A pragmatic method for deriving natural key spacing involves dividing the keysheet into the required number of naturals using a canted modular ruler. In general, this implies that

[96] The angle can be calculated as 89.2° = arctan(37/0.5).

front and back keyspacing were derived through independent means. Keyspace (i.e. keywell width) is also likely to have been used as the generating dimension for the case geometry of a perpendicular-stringed instrument.

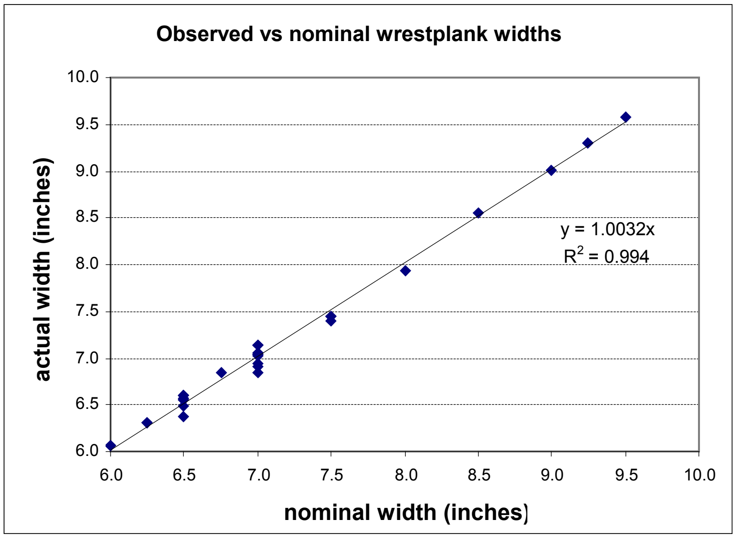


Figure 7. *Observed vs nominal wrestplank widths.*

Table 6. *Wrestplank widths expressed in builders' modular inches*

	Werkzoll	wrestplank width (front to back)			
		bass		treble	
	Werkzoll mm	nominal inches	observed inches	nominal inches	observed inches
Cristofori 1722	27.56	6	6.06	6	6.06
Späth und Schmahl ca 1770	26.33	6.25	6.30	6.25	6.30
Cristofori 1726	27.56	6.5	6.60	6.5	6.60
Langerer 1793	26.72	6.5	6.55	6.5	6.55
JL Dulcken ca 1795	26.00	6.75	6.85	7	6.85
Walter ca 1795	26.33	7	6.95	6.5	6.38
Walter ca 1800a	26.33	7	7.03	6.5	6.57
Walter ca 1800b	26.33	7	6.91	6.5	6.49
Ruckers 1637 AR	27.71	7	7.04	7	7.04
Ruckers 1679 IC	27.71	7	7.15	7	7.15
JD Schiedmayer ca 1794	26.33	7	7.06	7.5	7.44
Hofmann 1800	26.33	7.5	7.44	7.5	7.41
Streicher 1814	25.94	8.5	8.56	8	7.94
Fritz ca 1825	26.33	9.25	9.30	9.25	9.30
Graf 1826	26.33	9.5	9.57	9	9.00

3. **Structural case width for parallel instruments.** For the instruments in this study strong evidence has been found that the structural case width was derived as a nominal half modular inch dimension. This width is the generating dimension for a parallel-stringed instrument, and consequently would have had to be incorporated into the design at the earliest stages of construction.
4. **Long case dimensions and geometry.** Many long case dimensions are likely to have been direct modular measurements, but these can easily be confused with irrational dimensions that were the result of direct constructive geometry. An analysis of possible geometric layout methods can help to determine the extent to which modular measurement may have been used in the design, and which long case dimensions are likely to have been measured rather than constructed geometrically.
5. **String lengths.** For early instruments such as harpsichords and virginals there is reasonable historical source evidence to support the view that nominal string lengths which defined the string scaling were often modular dimensions. Observed string lengths may be seen to be consistent with a proposed modular inch, but they are often not accurate enough to provide useful quantitative input. Separate knowledge about a builder's likely scaling methodology is necessary to interpret these results. Piano builders may have used a proportional method for string scaling which did not involve nominal modular lengths.
6. **Small case dimensions.** The differences between nominal modular measurements used for small arbitrary case dimensions (less than 12 inches) are generally not large enough to be distinguishable from random errors. However, in some cases critical, but still arbitrary, small dimensions may be used to support a proposed inch size. The most important aspect of analysing small case dimensions is that they provide evidence that modular measurement was indeed used by builders for arbitrary design dimensions, sometimes in subtle ways to achieve a specific design objective.

LESSONS LEARNED

Three working 'groundrules' for the examination of an historical instrument were proposed recently:[97] '(i) analysis should be based on points or lines actually on the instrument; (ii) the geometry, arithmetic, or mensuration, should accord with theories, practices, or standards that were prevalent in the maker's own culture; (iii) the analysis should be repeatable, for example on several instruments of different models by the same maker.' Some discussion of these points may be useful in light of the material we have presented in these two articles. The meaning of the first 'groundrule' is unclear: Are these 'points or lines actually on the instrument' meant to be construction marks? If so, it is likely that most of these will not have survived or not be visible, therefore little significance can be attributed to their absence. Alternatively, the meaning may also be taken that only 'points and lines' in the body of the instrument are valid for use in a proposed layout procedure, but this is clearly contradicted by the bentside construction for Arnaut's harpsichord.[98] The third 'groundrule' is illogical as stated. While it is likely that the same design would have been used for nominally similar instruments of a particular builder, the requirement that it be applicable to different models makes no sense. In fact, a different design geometry is perhaps one of the main factors which defines a model as being 'different' from another. For instance, there is no *a priori* reason to suppose that the layout schemes used for Ruckers single and double manual harpsichords were necessarily related.

The second 'groundrule' has the most valuable content, but needs to be more focused. For example, placing any relevance on 'theories' and 'scholarly sources' will very quickly lead the

[97] Koster, 1998, op. cit., p.1.
[98] Henri-Arnaut de Zwolle, op. cit.

analyst astray. ‘Standards’, too, have been shown to be somewhat ephemeral and relate mostly to an individual builder. As for ‘practices’, this is certainly the key to a correct analysis of historical design and construction, but great care is necessary to ensure that practice is approached from a strictly historical perspective. This is essential to avoid the false conclusions which result from *ad hoc* assumptions that may seem simple and obvious, but are actually anachronistic and invalid in an historical context. Some organological problems that can be related to such false starts have been discussed: (i) use of colloquial dimensions as precise measures to support metrological conclusions, leading to complicated theories such as the ‘small’ and ‘large duimen’ supposedly used by the Ruckers; (ii) over-emphasis on tenuous metrological connections between builders and the locales where they worked; (iii) failure to consider direct constructive geometry as an alternative to modular measurement; (iv) failure to distinguish between irrational dimensions and their rational approximation; (v) reversal of the builder’s causality which generates keyspacing from keyboard width; (vi) critical reliance on information extraneous to the particular instrument being analysed; (vii) failure of abstract proportional analysis to identify those special relationships which are the direct consequence of the geometric construction actually used by a builder; (viii) a reductionist over-emphasis on details at the expense of a proper holistic view of a design in terms of modular geometry.

In abstract terms, fabrication of any artefact requires three processes: a design, its storage in some form, and its repeated reconstruction in the objects being made. A design based on geometric relationships can be constructed repeatedly any number of times on objects being made, with consistently predictable results. No permanent record of the design is necessary, since it is embodied in the geometric instructions which are so easily passed on by oral tradition. A master could adapt a design if required, inventing a new geometric ‘recipe’ specified as a series of steps to be followed. Success in design relied on sound practical experience and intuition, rather than mathematical ‘correctness’ and theoretical reasoning. By considering that these geometric recipes were particular to specific craftsmen, places, and times, the emergence of different regional schools of building can be anticipated. Unless a Guild-like system is responsible for setting strict guidelines which must be adhered to, geometric technique is likely to vary in detail between builders working in the same tradition, or even within the output of a particular builder. A maker may have regarded the details of his geometric designs to some extent as ‘trade secrets’, although none of them seems to have been able to keep these secret for very long, perhaps because the general practice was so well known and the details were so easy to learn and remember.

From the modern perspective it would probably have been more natural to ask ‘Why didn’t the historical maker *use* drawings?’ instead of the question posed in the titles of these articles. We have seen that the concept of reference drawing is actually so alien to the historical context that it might just as well be asked why the historical maker didn’t use a computer. Of course necessity may be the mother of invention, but in this case the historical builder didn’t use drawings because he didn’t *need* to use them, hence our question: ‘Why?’ Even Hubbard was not able to separate himself from his modern intellectual apparatus on this point: ‘Modern makers rely heavily on drawings and templates to speed up the process of construction. The high level of production which the old shops were able to attain without these aids is remarkable.’[99] To paraphrase Hubbard, the historical maker would probably have said something like the following about his modern counterpart: ‘Historical makers rely on proportional geometry to achieve a speedy and efficient process of construction with a high level of production from their shops. Modern makers do not appear to understand the principles that generate our designs, so they must work more slowly and rely heavily on their drawings.’ To understand truly the design of an historical instrument, it must necessarily be viewed in the same geometric terms as those of the original builder. This implies that authentic copies cannot be made by reproducing the sterile dimensions of an extant instrument using modern methodology, no matter how accurately this is done, and, in particular, truly historical instruments cannot properly be made by a builder using a design

[99] Hubbard, op. cit., p.215.

methodology based on the technical drawing. We have attempted to demonstrate that historical working practice need not be mysterious or secret. The basic principles are easy enough to deduce and can be applied to the practice of making stringed keyboard instruments with some confidence. What emerges is an efficient methodology, fundamentally different from modern practice, in which the drawing has no place. Perhaps one should not ask why they worked without drawings, rather why we choose to work with them.

APPENDIX. ***NOTES ON THE INSTRUMENTS AND ACKNOWLEDGEMENTS.***

DIMENSIONAL DATA IS GIVEN IN MM WHERE NO UNIT IS STATED. INSTRUMENT WAS MEASURED BY THE AUTHORS UNLESS THE DATA SOURCE IS OTHERWISE ACKNOWLEDGED BELOW.

Ruckers 1640a AR. Yale. Based on drawing by RK Lee, 1971 (with scale). Keyboard dimensions unavailable due to ravalement. Keyspace calculated from keywell assuming 1 inch (55 for two) keyblocks as in Ruckers AR1640b AR is consistent with nominal expected dimension.

Ruckers 1637 AR. GNM Inv. No. MIR 1073. Re-constructed original keyboard. Stringband 611 for 44 spaces, but uneven and first and last spaces are larger; distance 582 for 42 spaces. Wrestplank width is given without the protruding top veneer. A single-manual harpsichord in essentially original condition. ¾ inch keyblocks.

Ruckers 1679 IC. Washington, Smithsonian Cat. No. 75/31. Based on drawing by S. Germann & J. Scott O'Dell, 1975/77 (with scale). Stringband 668 (48 spaces). The only extant Ruckers single manual harpsichord with original chromatic compass.

Cristofori 1722. Museo Nazionale Strumenti Musicali, Rome. Data kindly supplied by Kerstin Schwarz. Stringband width 661 (48 spaces) implies allowance of 24 inch with inch = 661/24 = 27.54. A reported Florentine soldo is given as 27.56 by O'Brien. Spacing is compressed locally in groups of seven notes, between each of which is located a gapspacer (no dumb choir) with extra-wide space. No keyboard data available except three-octave span. Observed keysheet calculated from reported three-octave span (assuming all natural keys same width) is consistent with nominal keyspace of 25 inch (689) with clearance 2 mm top and bottom.

Cristofori 1726. Musikinstrumenten-Museum der Universität, Leipzig. Data kindly supplied by Kerstin Schwarz. Stringband width 661 (48 spaces) implies allowance of 24 inch with inch = 661/24 = 27.54. A reported Florentine soldo is given as 27.56 in O'Brien. Spacing is compressed locally in groups of seven notes, between each of which is located a gapspacer (no dumb choir) with extra-wide space. Three-octave span measured. Top and bottom natural keys extra-wide (25mm). See text for discussion of keysheet dimension and three-octave span.

Ruckers 1640b AR. Namur Hotel du Croix. Transposer. Data based on drawing by J Tournay 1972 & W Jurgenson 1989 on mylar (no scale). No keyboard data as keyboards are missing. Original disposition and registers.

Cristofori 1720. Metropolitan Museum Art, New York. Data kindly supplied by Kerstin Schwartz. Original compass FF-c3. Stringing data is not original due to heavy alterations. No keyboard data available except three-octave span. Observed keysheet calculated from reported three-octave span (assuming all natural keys same width) is consistent with nominal keyspace of 27.5 inch (758) with clearance 2 mm top and bottom. Nominal stringband width (53 spaces) is 26.5 inch (730).

G. Silbermann 1749. GNM Inv. No. MI 86. Data based on drawing by Antoine Leonard, 1984.Transposing instrument FF-e3 with additional EE choir. Keyboard is FF-e3.Stringband FF-e3 60 notes, or 59 spaces, is 787, giving inch of 2(787/59) = 26.68. Stringband EE-e3 is 801 = keyspace. Keywell width excludes 3mm visible veneer on each side. 13 mm space available for transposing. Case built like a harpsichord - no inner frame.

Stein 1783a. Private ownership, Germany. Signed and dated 1783. Phase II Stein (according to Latcham's terminology). Stringband width at the damper rack is 791 for 60 spaces (26.33 inch is 790). Keyframe is not rectilinear (862 back; 857 front).

JD Schiedmayer c.1794. GNM Inv. No. MIR 1102. Stringband FF-f3 at 791 gives inch of 791/30 = 26.37. Four octaves on the stringband (not including the slightly compressed bass strings) at 633 gives an inch of 26.38.

Walter pre-1776. Haydn-Haus, Eisenstadt. Signed but undated. Latcham dates this as *c.*1782. On the basis of the larger modular inch, a date of pre-1776 may be more appropriate. Data kindly supplied by Robert Brown. Stringband FF to f3 measured on the damper rack is 815; at key tails is 809. Spacing is locally consistent with Walter's normal working practice with one extra line shift for each of the wide Ds. Nominal stringband width with 26.70 inch at 30 5/12 inch = 812. Spacing also confirmed from Latcham nutpin data (see text). Keywell is not rectilinear (898 at back).

Walter c.1784. GNM Inv. No. MIR 1098. Signed but not dated. Keyboard data kindly supplied by Klaus Martius. Stringband FF-f3 is 802, consistent with one line shift for each of five wide Ds. Nominal stringband width is 30 5/12 inch = 802 with Viennese inch of 26.33. Additional data from Gerard Tuinman.

Späth und Schmahl c.1770. Tangentenflügel, Regensburg, label missing, *c.*1770 by dendrochronological dating. WLM Inv. no. 9.315. Earliest extant example. Exact replica 1993 by W Jurgenson on display. Extra-wide FF and f3 keys (1mm each). Stringband FF-f3 at 790, an allowance of 30 inch, pinned equidistantly with 61 spaces (including gapspacer dumb choir), can be derived from canted 30.5 inch over 30 inch stringband. Keytail spacing is narrower than stringband/damper spacing, to allow extra wide space (23) for damper lifter, which passes through the keypanel at the gapspacer. Keytail spacing can be derived separately for left and right half keyboard using canted 31 inch over 30 inch allowance: 395(30/31) = 382 per half, gives nominal central space = 790 – 2(382) = 25 consistent with observed width (23). In effect this is a variation on Stein's method which constructed a smaller derived inch unit to accommodate the gapspacer without changing stringband width.

Stein 1783b. WLM Inv. No. G4185. Signed and dated 1783. Stringspacing measured across the damper rack is 713 for 55 slots, giving inch of 25.93. Keyframe not rectilinear (back 855; front 857). Keyspace not rectilinear (817 back; 815 front). 2W = 927 at the front of the keywell where it has shrunk inward. 2W given is measured at the bellyrail.

Attr. Stein c.1785. WLM Inv. No. 1989-160. Attributed to Stein. (Fake?) label and undated. Stringband FF-f3 with one dumb choir 790 for 61 spaces gives inch of 25.91.

Langerer 1793. Finchcocks, UK. Data based on drawing by Chris Nobbs (with scale). Stringband FF-f3 with dumb choir is 815 for 61 spaces, giving inch of 26.72. Three-octave span not shown. 'Observed value' is calculated from octave span (162.5).

Hofmann 1800. Private ownership, The Netherlands. Drawing by Jos Noorhoff, The Hague, 1992. Gapspacer occupies width of 1 choir, but no dumb choir is pinned. Stringband FF-f3 is 790 for 61 spaces, giving inch of 25.95. Two keywell liners nominally at 1 inch each. Bass liner is thin at 11/12 inch, suggesting 2W was the nominal measured dimension.

Walter c.1795. Private ownership, Czech Republic. Signed but undated. Data kindly supplied by Paul McNulty. Stringband FF-g3 63 choirs is 827. Nominal 62 spaces at half-Viennese inch plus 5 lines (5/12 inch) is 827.

Walter c.1800a. KHM Inv. No. SAM 539. Signed but not dated. Data based on drawing by Karl-Heinz Hug, 1985 (with scale). Drawing is ambiguous on details of keyframe and keywell. Stringband 63 choirs is 827. Nominal 62 spaces at half-Viennese inch plus 5 lines (5/12 inch) is 827. Wrestplank width estimated because front veneer thickness is not shown.

Walter c.1800b. GNM Inv. No. MinE 109. Signed but not dated. Data based on drawing by Susanne Wittmayer, 1974 (with scale). Stringband 63 choirs is 827. Nominal 62 spaces at half-

Viennese inch plus 5 lines (5/12 inch) is 827. Bass liner (26) and over-thick treble liner (29) nominal 1 inch giving reduced keywell dimension compared to nominal, suggest 2W was the nominal measured dimension (and this was 3 mm shy of nominal as well)?

JL Dulcken c.1795. Smithsonian Institution, Inv. No. 303,537. Drawing by T Wolf et al, 1975 (with scale on main drawing, but not on keyboard drawing). Stringband measured dimension is 804 for 62 spaces, giving inch of 25.91. Keyframe is not rectilinear (front is 874). Outer case rests on the bottom –case width is interpreted as dimension to outside of liners.

JL Dulcken c.1798. WLM Inv. No. G4148. ca 1798 (undated). One gapspacer with dumb choir. Stringband of 68 choirs plus one gapspacer = 68 spaces is 886; random 56 spaces measured at 728, giving inch of 26.00. Keyspace (910 back, 915 front) and keysheet (911 front, 903 back) are not rectilinear. Keyframe and keywell dimensions are almost identical (no keyboard shift).

Streicher 1805. Private ownership, Germany. On loan to WLM. Signed No.649 and dated 1805. Two English style gapspacers with no dumb choirs. Stringspacing (and consequently keytail spacing) is consistent with 26.33 inch. Keys sawn front to back with wide Ds, and kapsels bent to align hammers to even spacing (i.e. no wide Ds at stringband). Choirs either side of the gapspacers are moved left and right by half a choir width. Otherwise gapspacers cause no irregularities in spacing. Stringband FF to c4 measured across the damper rack is 882. Nominal 67 spaces (for 68 choirs) is 33.5 inch = 882 with inch of 26.33. Keyframe is not rectilinear and has been planed on the treble side (942 at back). Space allotted in the case for keyframe is 948. Soundboard not original.

Walter und Sohn c.1810. WLM Inv. No. G8591. Signed and undated. Stringband spacing is locally variable, but consistent with nominal Viennese half-inch spacing over octaves. Three gapspacers do not cause direct spacing irregularities. Keywell is not rectilinear (back is 990).

Streicher 1814. WLM Inv. No. WEI 26 W6. Signed No.1060 and dated 1814. Stringband FF-f4 with two dumb choirs is 960 measured across the damper rack (74 spaces) giving inch of 25.94. Keywell width is to spruce liners, i.e. ignoring the 10mm of oak veneer keywell linings in bass and treble.

Streicher 1808. GNM Inv. No. MIR 1117. Signed No.764 and dated 1808. Data kindly supplied by Klaus Martius. Report by Michael Latcham also consulted, and personally inspected and measured. Stringband CC-f4 is 1026, 79 spaces = 1024, giving inch of 25.93. Keyspace calculated from reported three-octave span plus 2mm clearance top and bottom.

Dieudonne & Schiedmayer 1815. WLM Inv. no. G4339. English action, so no damper rack. Four English style gapspacers with no extra spacing allowance and minimal local cramming to fit. Stringband measured C to f4 is 66 spaces and 870, giving inch of 26.36.

Dieudonne & Schiedmayer c.1820. Private ownership, Germany. Undated. Viennese action. One gapspacer between e1 and f1, no dumb choir but extra-large space of 23+ mm allowed. Observed stringband width CC-f4 of 1026 is 39 Viennese inches of 26.33, nominally 1027. 78 choirs, or 77 evenly-sized spaces at Viennese half-inch + additional space at central gapspacer gives nominal stringband width of 1024. Outer case nominally ½ inch (observed 14 spine, 15 cheek). Three-octave span and keysheet width calculated from observed keyspace and 1mm clearnace top and bottom.

Graf c.1825. WLM Inv. No. 1986-283. Signed Op 513 and undated. One gapspacer with dumb choir.

Graf 1826. Vleeshuis Museum, Antwerpen. Signed Op 995 and dated 1826. Data based on drawing by C Clarke, 1998 (with scale). Stringband CC-f4 with one dumb choir is 1029 for 78 spaces, giving inch of 26.38 (Viennese is 1027 nominal). Keyblocks 3/4 inch bass and 3/4+ inch treble. Liners 5/4 inch bass, 5/4+ inch treble.

Fritz c.1825. Private ownership, The Netherlands. Signed and undated. Stringband 1053 (1053 is 40 Viennese inches). Data kindly supplied by Paul Poletti. Three-octave span calculated from observed keyspace and 2mm clearance top and bottom.

Graf c.1830. Private ownership, USA. Signed Opus 2148 and undated. Data kindly supplied by Penelope Crawford. One gapspacer with dumb choir.

Boisselot et Fils 1845. In the possession of one of the authors. Signed No 2232 and undated. Stringband of 1125 has three dumb choirs for three gapspacers, plus extra treble choir above top note, i.e. 84 choirs, or 83 spaces. 1125 = 83/2 inch with Paris inch of 27.08. Spacing confirmed over various string spans. 2W dimension is to outside of inner frame. Keywell is 2W less two half-inch liners and two lining veneers (3 mm total).

Hatzenbuehler c.1845. In the possession of one of the authors. Upright piano. Signed No.1258 and undated. At least after 1841. Stringband measured at the strikeline. Lowest 10 bass string choirs (9 spaces) are compressed by ½ inch at the hitchpin end into a space of 4 inches (108).

Erard 1836. In the possession of one of the authors. Signed No.13918, Paris. Four of five iron over-soundboard braces with gapspacers require space of 4 string choirs. 82 string choirs. Total stringband width of 1152 is 42.5 inch (1151) required for 86 choirs, or 85 spaces, with Paris inch of 27.08. Outer case and liners are integral so case width 2W is really outer case less veneers.

Appendix. Dimensional data for 35 instruments referred to in analyses in the text.

			(1)	(2)	(3)	three-octave span(4)		keysheet(5)			keyspace			keyframe			keywell			structural case width 2W			outer case width			wrestplank width					
																										bass			treble		
	mod. inch	keybd comp.	N	gs	nat	(calc.) or obs.		nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom
	mm					mm	inch	in	mm	mm	in	mm	mm	in	mm	mm	in	mm	in	mm	mm	in	mm	mm	in	mm	mm	in	mm	mm	in
Ruckers 1640a AR	27.71	C/E-c3	45		27						23.5	(649)	651				25.5	704	707	26.5	731	734	26.5	731	734						
Ruckers 1637 AR	27.71	C/E-c3	45		27	499	18.00		646	640+	23.5	650	651				25	690	693	26	718	720	26	718	720	7	195	194	7	195	194
Ruckers 1679 IC	27.71	C-c3	49		29	499	18.00		691	688+	25	693	693				26.5	732	734	27.5	760	762	27.5	760	762	7	198	194	7	198	194
Cristofori 1722	27.56	C-c3	49		29	497	18.03		(685)	685	25	(689)	689				25.5	702	703	28	772	772	29	794	799	6	167	165	6	167	165
Cristofori 1726	27.56	C-c3	49		29	491	17.82		682	677+	25	684	689				26.5	732	730	28	778	772	29	801	799	6.5	182	179	6.5	182	179
Ruckers 1640b AR	27.71	C/E-f3	50		30	499	18.00			712+	26	721	720				28	770	776	29	800	804	29	800	804						
Cristofori 1720	27.56	C-f3	54		32	496	18.00		(755)		27.5	(759)	758							32.5	898	896	34	938	937						
G. Silbermann 1749	26.70	FF-e3	61		35	482	18.05	30	801	801		805		32.5	866	868	35	935	935	36	962	961		N/A							
Stein 1783a	26.33	FF-f3	61		36	475	18.04				31	818	816		862		33.5	880	882	35.5	935	935	36.5	964	961						
JD Schiedmayer c.1794	26.33	FF-f3	61		36	475	18.04		815		31	818	816	33	868	869	33.5	883	882	35.5	940	935	37	971	974	7	186	184	7.5	196	197
Walter pre-1776	26.70	FF-f3	61		36	485	18.16	31	829	828		831		33.5	894	894	33.5	896	894	36	959	961									
Walter c.1784	26.33	FF-f3	61		36	477	18.12	31	815	816		819			873			875		36	950	948	37	974	974						
Späth und Schmahl c.1770	26.33	FF-f3	61	1	36	478	18.12		820	818	31.5	822	829	33.5	882	882	34.5	906	908	36	946	948	37	971	974	6.25	166	165	6.25	166	165
Stein 1783b	25.93	FF-f3	61	1	36	474	18.29		812		31.5	817	817	33	857	856	34	880	882	36	932	933	37	960	959						
Attr. Stein c.1785	25.91	FF-f3	61	1	36	(471)	18.19		807		31.5	812	816	33	854	855		915		35.5	923	920	37	955	959						
Langerer 1793	26.72	FF-f3	61	1	36	(488)	18.24		835		31.5	842	842					915	913	35.5	949	949	36.5	973	975	6.5	175	174	6.5	175	174
Hofmann 1800	26.33	FF-f3	61	1	36	482	18.31		824		31.5	828	829	32.5	856	856		886		35.5	936	935	36.5	962	961	7.5	196	197	7.5	195	197
Walter c.1795	26.33	FF-g3	63		37	479	18.19	32	843	843					887					36.5	962	961				7	183	184	6.5	168	171
Walter c.1800a	26.33	FF-g3	63		37	480	18.23	32	844	843		848			899		35	922	922	37	974	974	38	1001	1001	7	185	184	6.5	173	171
Walter c.1800b	26.33	FF-g3	63		37	480	18.23	32	843	843		847			889		35	913	922	37	971	974	38	1001	1001	7	182	184	6.5	171	171
JL Dulcken c.1795	26.00	FF-g3	63	1	37	(472)	18.16	32	(831)	832		834			878		35	913	910	37	962	962	38.33	998	997	7	178	182	7	178	182
JL Dulcken c.1798	26.00	FF-c4	68	1	40	478	18.38	35	911	910		915					37	963	962												
Streicher 1805	26.33	FF-c4	68		40	474	18.00		902		34.5	907	908	36	948	948	36.5	964	961	38.5	1018	1014									
Walter und Sohn c.1810	26.33	FF-f4	73		43	480	18.23		985		37.5	987	987		1033		39.5	1039	1040	42.5	1118	1119	43.5	1148	1145						

						three-octave span		keysheet			keyspace			keyframe			keywell			structural case width 2W			outer case width			wrestplank width bass			wrestplank width treble		
	mod. inch	keybd comp	N	gs	nat	(calc.) or obs.		nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom	nom	obs	nom
	mm					mm	inch	in	mm	mm	in	mm	mm	in	mm	mm	inch	mm	in	mm	mm	in	mm	mm	in	mm	mm	in	mm	mm	in
Streicher 1814	25.94	FF-f4	73	2	43	474	18.27		969		37.5	973	973	39	1011	1012	41	1063	1064	44	1138	1141	45	1166	1167	8.5	222	220	8	206	208
Streicher 1808	25.93	CC-f4	78	2	46	475	18.32		1039		40.25	1043	1044	41.5	1076	1076	43	1114	1115	45.5	1179	1180									
Dieudonne & Schiedmayer 1815	26.33	CC-f4	78		46	479	18.19		1049		40	1052	1053	42	1104	1106		1110													
Dieudonne & Schiedmayer c.1820	26.33	CC-f4	78	1	46	(479)	18.19		(1048)		40	1050	1053	41	1084	1080		1089			1179		46	1208	1211						
Graf c.1825	26.33	CC-f4	78	1	46	479	18.19		1049		40	1052	1053		1098		42	1104	1106												
Graf 1826	26.33	CC-f4	78	1	46	479	18.19		1049		40	1053	1053		1098		42	1107	1106	44.5	1176	1172	46	1208	1211	9.5	252	250	9	237	237
Fritz c.1825	26.33	CC-g4	80	1	47	(478)	18.16		1069		40.75	1073	1073	43	1132	1132		1150		46.5	1227	1224	48	1262	1264	9.25	245	244	9.25	245	244
Graf c.1830	26.33	CC-g4	80	1	47	(481)	18.26		1075		41	1078	1080				43	1132	1132	46	1208	1211	47.25	1244	1244						
Boisselot et Fils 1845	27.08	CC-g4	81	3	47	494	18.24		1106		41	1110	1110	43	1160	1164	44	1190	1192	45.5	1233	1232	45.5	1233	1232						
Hatzenbuehler c.1845	27.08	CC-g4	80		47	484	17.87	40	1083	1083																					
Erard 1836	27.08	CC-a4	82	4	48	497	18.35		1134		42	1137	1137				45	1222	1219	47	1270	1273	47	1270	1273						

(1) N = number of notes in the keyboard compass (keybd comp.)

(2) gs = number of gapspacers occupying extra string choir spaces

(3) nat = number of naturals on the keyboard

(4) Parentheses on 'observed' dimension indicates an indirect value calculated from observed or reported dimensions. See notes below for explanations.

(5) For all Ruckers instruments 'observed' three-octave span has been assumed to be 499 mm. Nominal keysheet dimension indicated '640+', '688+' etc are minimum dimensions to which any extra-width on top or bottom keys must be added.